Use of the Fresnel Transform to detect and correct field drift artefacts in MR images

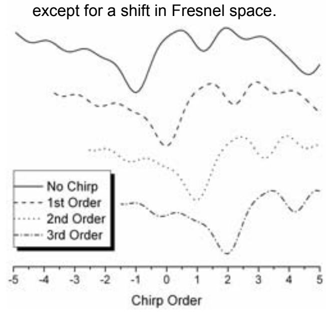
J. K. Alford[1], B. A. Chronik[1]

[1]Physics and Astronomy, University of Western Ontario, London, Ontario, Canada

Introduction: Magnetic field fluctuations cause difficulties in all disciplines of MR. These fluctuations result in phase noise capable of producing artefact ranging from blurring to severe image distortion. A linear drift in B_0 causes each isochromat to change slightly in frequency during *k*-space acquisition. This linear frequency change is generally referred to as a ‘chirp’. The Fourier-transform of a signal with chirp is distorted from that of a clean signal. In the absence of signal averaging, signal chirp produces artefacting similar to wrap around in the phase encoding direction. Averaging can reduce this effect, but doing so results in image blurring. If the chirp were exactly known the *k*-space data could be fixed before Fourier-transformation. Unfortunately, the rate of field drift is indeterminable; resulting from heat-induced changes in passive/resistive shims and gradients, feedback circuit instabilities and eddy currents. Systems using resistive magnets such as field-cycled MRI (FCMRI) are especially predominant due to field drift as a result of thermal expansion and power supply fluctuations.

A special transform was developed in order to deal with chirps. The transform uses a weighted Fresnel transform (WFT)[1] able to detect signal chirp in *any* complex dataset. If the main field drifts during a scan of *k*-space and then drifts by a different amount during a subsequent scan of the same row of *k*-space these two scans should not be averaged. They both contain phase errors, and even worse they contain different phase errors; however, the WFT transforms of each scan are identical except for a measurable, linear displacement in Fresnel space. Measuring the displacement indicates exactly the amount of chirp in the second signal with respect to the first. The difference in chirp can be easily removed by multiplying the second dataset by $exp(-2\pi it^2c)$ where c is the measured displacement. Ignoring random noise differences between scans; the two datasets are then identical. Such an operation removes the soft ‘blurring’ artefact mentioned above. If n duplicate rows of *k*-space are measured, each row can be ‘modified’ to be identical to some average ‘chirp’ rate of the group. Provided that average chirp rate is known *a priori*, elimination of the average chirp realigns the entire set of rows. All chirping has been removed and the dataset can be Fourier-transformed into an image.

Figure 1. The Fresnel Transform of a FID acquired with different levels of magnet chirp. The lines are identical except for a shift in Fresnel space.



Methods: Any repetitive signal can be represented as an infinite sum of sine and cosine functions having a period of p/n where p represents the period of signal repetition and n is an integer value (eq.1). This is recast in the more convenient exponential form given in eq.2. This equation is appropriate for describing the signal obtained from the sample within a static main field. If the magnetic field drifts *linearly* during signal acquisition a chirp is produced and the signal equation is modified to eq.3. Here chirp rate is written relative to the time of acquisition of the entire row of *k*-space. $c = 1$ is a chirp of order 1 and adds an extra 2π radians over the entire time of acquisition. This is equivalent to a 3ppm shift during a 20ms acquisition at 1.5T. Since the chirp is the same for each isochromat, the signal equation can be written as eq.4 which is nothing more than the original signal times a chirp: $s(t)*exp(2\pi ict^2/p^2)$. Applying the weighted Fresnel transform given by eq.5, to both the original dataset $s(t)$ and the chirping data dataset $s'(t)$ results in $S(\tau)$ and $S'(\tau)$. The result is that $S'(\tau) = S(\tau + c)$. The unknown phase noise has been transformed to a simple shift in Fresnel space.

$$s(t) = \sum_{n=0}^{\infty} a_n \sin(2\pi nt/p) + b_n \cos(2\pi nt/p) \quad \text{[eq.1]}$$

$$s(t) = \sum_{n=-\infty}^{\infty} c_n \exp(2\pi int/p) \quad \text{[eq.2]}$$

$$s'(t) = \sum_{n=-\infty}^{\infty} c_n \exp\left[2\pi i\left(nt/p + ct^2/p^2\right)\right] \quad \text{[eq.3]}$$

$$s'(t) = \left[\sum_{n=-\infty}^{\infty} c_n \exp(2\pi int/p)\right] \exp(2\pi ict^2/p^2) \quad \text{[eq.4]}$$

$$s'(t) = s(t)\exp(2\pi ict^2/p^2) \quad \text{[eq.5]}$$

$$S(\tau) = \int_0^p s(t) t \exp(2\pi ic\tau^2/p^2) dt \quad \text{[eq.6]}$$

The shift between $S(\tau)$ and $S'(\tau)$ can be easily measured and the datasets correlated by multiplying the corrupted signal $s'(t)$ by $\exp(-2\pi ict^2/p^2)$. This shifting is shown graphically in Figure 1. The top line is the weighted Fresnel transform of a simulated MRI free induction decay (FID). The second line is the WFT transform of the same FID acquired during magnet chirping. In Fresnel space these waveforms are identical except for a shifting in chirp order. The following lines show subsequently acquired FIDs subjected to different amounts of magnet drift during acquisition. This amount of chirp is quite large and is simply used as an example.

Results: A computer simulation shows the disastrous effect of chirping on MRI image quality (Figure 2a). The amount of chirp present on each line of *k*-space was chosen randomly within the range of chirp values anticipated in our FCMRI system[2]. Figure 2b shows that only limited artefact can be removed by averaging. The WFT method removes most of the chirp artefact from the signal (Figure 2c). Increased averaging would improve 2c, but only increase the blurring in 2b.

[1] Abramowitz *et al.*, Handbook of mathematical functions. Dover Pub., 1965.

[2] Gilbert *et al.*, Thermal modeling of resistive magnets for field-cycled MRI, MR Engineering, 2005, vol. 26b, page 55.

Figure 2.