Spatial-Spectral Holographic Interpretation of High Field MR Imaging : Analysis, Implications and Experiments at 7T

A. M. Kiruluta[1,2]

[1]Radiology, MGH, Boston, MA, United States, [2]Physics, Harvard University, Cambridge, MA, United States

Abstract: The fundamental MR imaging equation (1) can be derived from a generalized 3D spatial-spectral holographic wavefront reconstruction interpretation similar to that in quantum optics. A spatial-spectral holographic interpretation arises naturally in MR from an inhomogeneous linewidth broadening (~1/T2*) due to either an imposed set of linear orthogonal gradients fields or from the intrinsic chemical shift anisotropy. We can thus think of NMR k-space as a spatial-spectral holographic grating. The spatial holographic component arises from dielectric effects at high field strength (> 4T) when the excitation wavelength is less than or commensurate with the size of the imaging sample and the propagation wave vector $k = \lambda/2\pi$ cannot be neglected. In this work, it is shown analytically and experimentally that an inhomogeneously broadened NMR absorber can accomplish key spatial-spectral holographic characteristics such as storage, recall, time-reversal and matched filtering of gradient-weighted RF pulses in a three pulse stimulated echo experiment.

Methods: An RF excitation, coincident with an applied gradient field acts as a 'read-out' pulse with a spatial-spectral bandwidth that is 'Bragg-matched' to the grating vector $M(k_x,k_y,k_z)$ which thus diffracts into signals whose inverse Fourier transform is the desired image $m(x, y, z, \sigma)$ projected along the chemical shift axis σ, for each MR active chemical species in the sample. For a given chemical species, the 3D NMR stimulated echo imaging (STE) equation with linear gradient fields can be expressed in a holographic formulation as:

$$s(k(\vec{r},t)) = \int_v d^3r\rho(\vec{r})e^{-\phi(\vec{r},t)} = \int_v d^3r\rho(\vec{r}) \int_{-\infty}^{\infty} d\omega \left[E_3(\omega)E_2(\omega)E_1^*(\omega)e^{-i2\pi k(\vec{r},t)\cdot\vec{r}}\right] e^{-i\omega[t-(t_3+\tau)]}$$

where $E_j(\omega), j = 1 \cdots 3$ represents the Fourier decomposition of each of the respective excitation pulses in the STE sequence shown in Fig. 1 and assuming a laboratory frame of reference. Readout then represents a 'burning' of a spectral hole in the absorption spectrum of the spin resonant system. The gradients thus play the additional task of spectrally multiplexing the hologram read-out via slice selection and frequency/phase encoding.

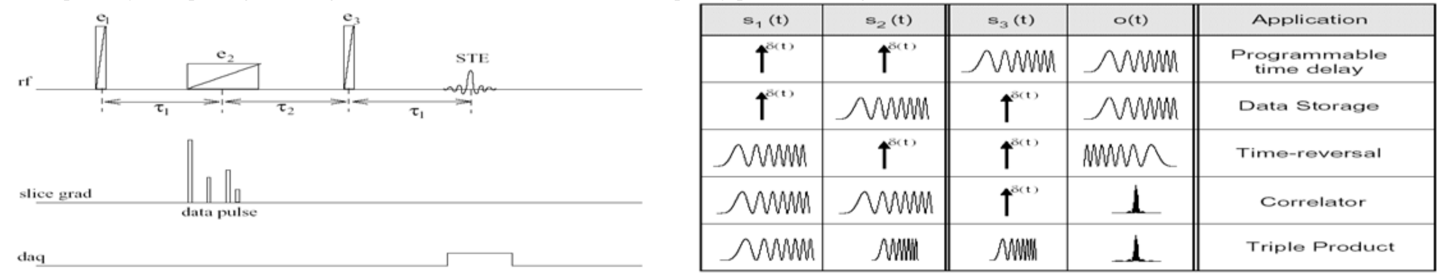

Figure 1: Timing diagram for storage, recall and filtering of RF pulses in an MR sample. The data pulses are applied as a slice selective gradient so that in combination with the chirped RF pulse e_2, result in a spectral hole in the medium. This data dependent spectral hole distribution is interfered with the population excitation of the first chirped RF pulse e_1 to create a spatial-spectral grating. The table shows permutations of potential processing functions from a combinations of these three pulse sequence. For example, if the 1st and 3rd pulses are very short then the output is a temporal replica of the 2nd pulse –storage. Without loss of generality, data pulses are shown as chirps but any crafted pulses can be used as long as their spectral content is within the inhomogeneous bandwidth of the medium which is on the order of 1/T2*.

Results:

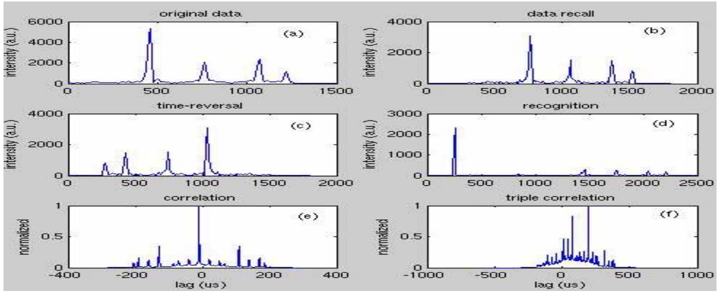

Figure 3: Storage, recall and matched filtering of a pulse consisting of four sub pulses in a three pulse stimulated echo experiment with an MR phantom. During the recording of the spatial-spectral grating, two pulses consisting of the reference (E_1) and the signal (E_2) spectrally interfere in the inhomogeneously broadened NMR absorber. (a) shows the original signal (data) pulses respectively, (b) if the reference pulse is applied first, then the free decay signal has the same time order as the signal pulse (storage) (c) if the signal pulse is applied before the reference pulse, then the signal pulse echoed from the spatial-spectral grating is time reversed (conjugate holography) (d) recognition (e) cross-correlation function is obtained from temporal matched filtering. (f) Triple correlation from all three pulses.

Conclusions: Fundamental properties of holography such as storage, recall and matched filtering have been experimentally demonstrated. Time delay and triple product processing are simple extensions of these fundamental properties. This holographic interpretation of NMR has implications on alternative spatial encoding schemes, new approaches to field homogeneity shimming using phase conjugate imaging principles as well as imaging using controlled gradients induced quantum beats in optical coherent transient materials.

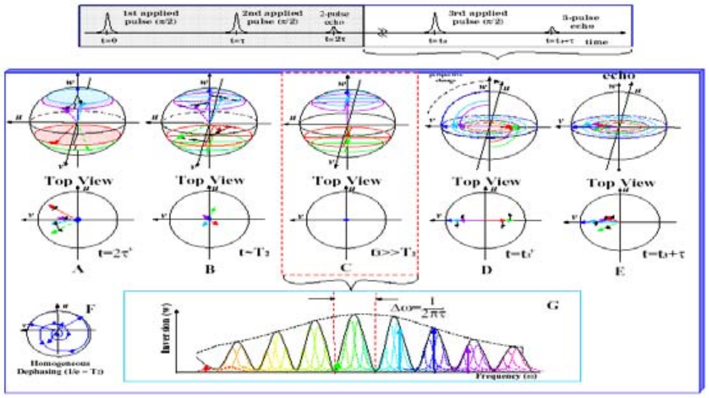

Figure 2: Application of the 3rd pulse in a STE sequence. Figure A-F illustrate the effect on the Bloch vectors for different resonance detunings as a function of time just after the two pulse echo at $t = 2\tau$ and up until the STE. The inset (G) shows a simulation of the spatial-spectral holographic grating after the second pulse stored as an inversion of the spin population.

References: [1] P. Mansfield and P. K. Grannell " NMR diffraction in Solids," J Phys C Solid State 6, 422-426 (1973).