Tumor Growth with Anisotropic Diffusion

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INTRODUCTION

Brain tumors such as gliomas show a very irregular pattern in their growth. A question that can be asked is whether the growth of tumors is modulated by the anisotropy of the brain tissue. Here we propose a mathematical and computational scheme using reaction-diffusion equations and Level-Set methods [1,2], for DTI data driven evolution of tumor fronts. The starting tumor front is obtained from contrast enhanced T1 MR images. We also compare the growth of a tumor driven by full anisotropic diffusion tensor with the growth driven by only the isotropic part of the diffusion tensor (trace ADC).

METHODS

We developed a reaction-diffusion model for the anisotropic growth of tumors. The reaction-diffusion equation was further transformed to a Hamilton-Jacobi form for a moving front [3]. Then the tumor front was evolved computationally from the given initial tumor segmented data obtained from contrast enhanced T1 images. Evolution of the tumor front is accomplished by a Level-Set scheme[1,2] using a robust Roe-Fix up-winding scheme [2] and fast re-initialization of the distance-transforms [1,2]. To keep the number of unknowns parameters as small as possible, we used a simple model that has only three parameters, the intrinsic growth rate of the tumor λ, a proportionality factor α, and an exponent *n* that determines how important the magnitude and shape of the water diffusion tensor is for anisotropic growth of tumors. A Hausdorff like robust measure can be used to minimize the difference between evolved fronts with the observed tumor front at future time points to determine the free parameters.

$$\mathbf{D}_{Tumor} = \alpha \mathbf{D}^n, \; \frac{\partial \rho}{\partial t} = \alpha \left(\nabla \cdot \mathbf{D}^n \cdot \nabla\right)\rho + \lambda\, g(\rho) \;, \; \frac{\partial G}{\partial t} = \alpha \left(\nabla G\right) \cdot \mathbf{D}^n \cdot \left(\nabla G\right) + \lambda \;\cdot$$

The first equation on left models the effective diffusion tensor for tumor cells as being proportional to a power of the water diffusion tensor. The key assumption is that the same brain tissue structural anisotropy that gives rise to anisotropic water diffusion may also drive the anisotropic part of tumor growth. The second equation is the reaction-diffusion model with as the tumor cell number density and g( ) as a logistic type growth function [3], **D** is the water diffusion tensor. The third equation is the corresponding Hamilton-Jacobi front equation for the front given by a level-set function *G*. By simple mathematical transformations we eliminated λ and α from the evolution equation. For the data shown in Fig.1 the exponent *n* was set equal to 2. The evolution under full diffusion tensor is anisotropic. The evolution under only the isotropic part (trace ADC) is isotropic.

RESULTS & DISCUSSION

A Level-Set method for evolving tumors based on tumor outlines obtained from contrast enhanced T1 images and diffusion tensors obtained from DTI data, is presented. Anisotropic evolution of tumors driven by full diffusion tensor creates more complex structures and tends to follow the white matter fibers more than a simple isotropic evolution. The question whether growing tumors follow existing tissue anisotropy, and to what degree, is a valid and open one. Although growing tumors may destroy and/or displace fiber tracts, introducing a complication. This method may be more applicable to infiltrating tumors than to those that cause displacement of the fiber tracts. The possibility that the complex growth patterns of tumors are affected by the anisotropy in the white matter is significant from a clinical point of view.

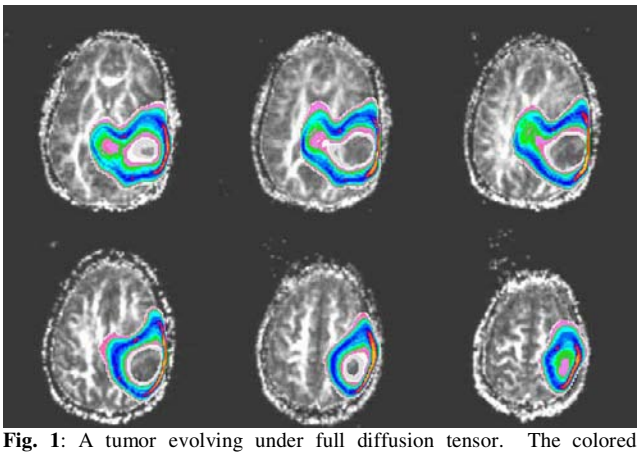

Fig. 1: A tumor evolving under full diffusion tensor. The colored contours show the evolving fronts at different times. The background images are Fractional Anisotropy (FA) maps. The uncolored central part of the contours shows the location of the original tumor. This anisotropic evolution creates more complex structure in the evolution and tends to follow the white matter tracts more closely, especially along the corpus callossum. The data were obtained from a GE Signa 1.5T MRI scanner. DTI data has 128×128 matrix, 1.72×1.72 mm^2 pixels., and contrast enhanced T1 data from which the original tumor shape was computed, has 256×256 matrix, 0.859×0.859 mm^2 pixels. Both datasets had 22 slices of 5 mm thickness with 1 mm gap.

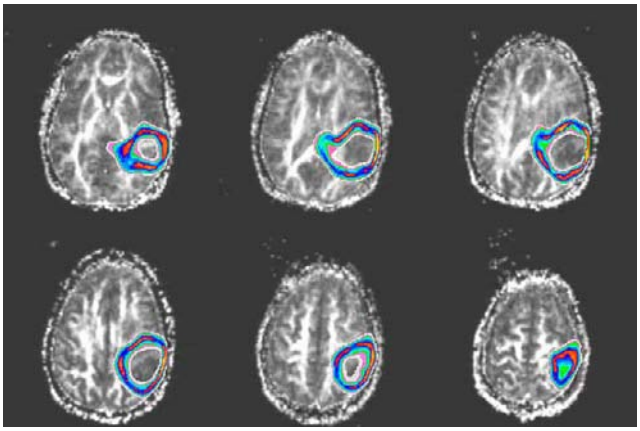

Fig. 2: The same tumor as in Fig.1 evolving under only the isotropic part (trace ADC) of the diffusion tensor. This isotropic evolution, creates less complex structure and shows weaker tendency to follow the white matter tracts.

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