# Adaptive SVD thresholding is shown to be more appropriate for partial brain scans (*TR* = 1 s) rather than full brain scans (*TR* = 2 s)

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## INTRODUCTION

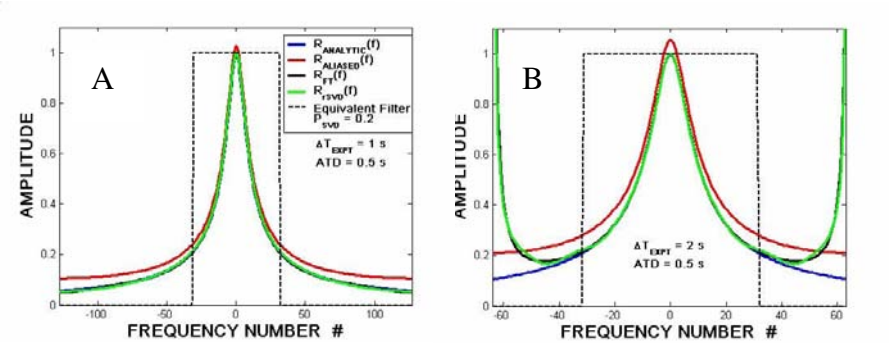
In magnetic resonance dynamic susceptibility contrast (DSC) perfusion studies, the cerebral blood flow (CBF) parameter is estimated from the peak of the scaled residue function obtained from deconvolving the tissue concentration curve $c_{VOI}(t)$ by the arterial concentration curve, $c_a(t)$ [1]. Noise reduction techniques are required to ensure singular value decomposition (SVD) and Fourier transform (FT) deconvolution stability; but these methods remove both high-frequency noise and signal components of the residue function. This distorts the estimate of the residue function and leads to incorrect CBF estimates [2]. Adaptive noise filter thresholds [3] have been suggested as a means of reducing these CBF biases. However, we have found limitations when applying adaptive filtering to full-brain scan studies with a DSC image sequence temporal resolution $TR \approx 1.85 - 2.25$ s. Extensive residue function distortion (aliasing) at high frequencies can be anticipated at such a low temporal resolution since the experimental conditions do not permit the Nyquist sampling criteria ($TR \approx 1 / 32$ s) to be met for the sharp edges present in the exponential and rectangular residue functions. These high frequency signal distortions would be theoretically expected to lead to severe limitations to the practical use of adaptive thresholding. Therefore a balance must be struck between adjusting the noise threshold through adaptive filtering to a low enough value to re-introduce higher frequency missing signal components while not reducing the level to such an extent as to introduce undesired signals distortions.

## METHOD

The vascular bed was considered as a single, well-mixed compartment $R_{ANALYTIC}(t) = exp(-t / MTT);\ t \geq 0$ [1]. The arterial input function was modeled by a gamma-variate function [1]. Tissue signal samples were analytically generated at times $t = n\ \Delta T$, $0 \leq n < N$, with the sample interval $\Delta T = TR$ equal to $\Delta T = 1$ s and 2 s. The true frequency components of the residue function were calculated, $R_{ANALYTIC}[m\Delta f]; 0 \leq m < N$, $\Delta f = 1/N\Delta T$. The aliased $R_{ALIASED}[m\Delta f] = DFT\{ R[n\Delta T] \}$ were obtained, where $DFT\{ \}$ is the discrete Fourier transform. The spectral components $R_{FT}[m\Delta f] = DFT\{R_{FT}[n\Delta T]\}$ and $R_{rSVD}[m\Delta f] = DFT\{R_{rSVD}[n\Delta T]\}$ were calculated from the residue functions obtained using two delay-insensitive deconvolution algorithms – Fourier transform (FT) [1] and reformulated SVD algorithm [2] respectively.

## RESULTS

The values for $R_{ALIASED}[m\Delta f]$ (red), $R_{rSVD}[m\Delta f]$ (green) and $R_{FT}[m\Delta f]$ (black) are compared to the true spectral components of the residue function $R_{ANALYTIC}[m\Delta f]$ (blue) for simulated (Fig. A) partial brain scans ($TR \approx 1$ s) and (Fig. B) full brain scans ($TR \approx 2$ s). The spectral distortions introduced by under sampling $R_{ANALYTIC}(t) = exp(-t / MTT)$ can be seen by the differences between $R_{ALIASED}[m\Delta f]$ and $R_{ANALYTIC}[m\Delta f]$ for all frequency components and values of $TR = \Delta T$. However, distortions of the residue function obtained from deconvolution are not prominent for $TR \approx 1$ s as it can be seen that $R_{ANALYTIC}[m\Delta f] \approx R_{rSVD}[m\Delta f] \approx R_{FT}[m\Delta f]$ for both rSVD and FT algorithms. For $TR \approx 2$ s, however, severe distortions of the residue function spectral components occur just past the end of the implicit filter (dotted box) corresponding to an adaptive SVD threshold of $P_{SVD} = 0.2$ [2, 4]. These simulation results imply that the use of adaptive filtering is appropriate over a wide range of filter thresholds for partial brain scans ($TR \approx 1$ s). For this *TR* value, moving the noise threshold does not introduce any significant signal distortion. However for full brain scans ($TR \approx 2$ s), the range of adaptive filtering is essentially limited to the equivalent of the standard SVD threshold, $P_{SVD} = 0.2$ [1].




Spectral components for the true (blue), aliased (red) and deconvolved (FT-black, rSVD-green) residue functions for a simulated (A) partial brain scan ($TR = 1$ s) and (B) full brain scan ($TR = 2$ s) with the equivalent frequency bandwidth of a $P_{SVD} = 0.2$ SVD threshold superimposed (dotted line). Full application of adaptive thresholding is possible with the partial brain scan, since little distortion is present, but only limited application is possible with the full brain scan.

## CONCLUSION

Adaptive thresholding techniques have been suggested to minimize the error in CBF estimates introduced when removing high frequency signal and noise components while invoking noise stability criterion during FT and SVD deconvolution. Valid results will be obtained when applying an adaptive threshold only if signal distortions are not introduced when moving the threshold. We have demonstrated why adaptive thresholding is appropriate over a wide range of thresholds for partial brain scans ($TR \approx 1$ s). However, for full brain scans ($TR \approx 2$ s), the adaptive threshold range is essentially limited to the equivalent of the standard $P_{SVD} = 0.2$ [1] in common use. After application of an adaptive threshold, a truncated spectral estimate of the residue function spectral components remains. Frequency modeling techniques [*e.g.* 5, 6] are then appropriate to recover the missing spectral components to provide less distorted residue functions and hence improved CBF estimates.

**REFERENCES**

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