FMag - Fast Computation of Static Magnetic Field Perturbation due to Inhomogeneous Media

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Introduction Static magnetic field inhomogeneities generate multiple phenomena in magnetic resonance imaging (MRI). They are the source of contrast in essential MRI methods such as fMRI, FLASH and other gradient echo sequences. As a result of T_2^* mechanisms and magnetization phase accumulation, they are also the cause of geometrical distortion and attenuation of intensity. B_0 inhomogeneities result from perturbation caused by the magnetic permeability (susceptibility) of the organ under investigation rather than imperfection in magnet manufacture/assembly. The simulation of B_0 field is important in order to understand the mechanisms of T_2^* contrasts in fMRI and GE studies, and to assist in research aimed at the reduction of inhomogeneity – related artifacts by the introduction of hardware for better shimming of the imaged organ by static [1,2], and active shims or computational methods combined with sequence manipulation.
Fast, reliable algorithms for B_0 simulation are vital for any of the above applications. In the current work, the suggested algorithm (FMag) yields a more accurate result, compared to a commercial software and does so more than an order of magnitude faster.

Methods In simply connected domains devoid of electric currents, the magnetostatic problem of determining the $\mathbf{B}_o$ field within the organ or tissue of interest can be formulated in terms of magnetic scalar potential. Note that the magnetic field (flux density) can be derived from the scalar magnetic potential $\Phi(\boldsymbol{r})$ as $\mathbf{B}_o(\mathbf{r}) = -\mu(\mathbf{r})\nabla\Phi(\mathbf{r})$, where $\mu(\mathbf{r})$ denotes the magnetic permeability of the body at point $\mathbf{r}$. The scalar potential can be shown to satisfy integral equation

$$\Phi(\mathbf{r}) = \Phi^{\mathrm{i}}(\mathbf{r}) + \int_V G(\mathbf{r},\mathbf{r}')\left[\nabla\cdot\left(\frac{\mu(\mathbf{r}')-\mu_a}{\mu_a}\nabla\Phi(\mathbf{r}')\right)\right]dv' \qquad (1)$$

where $G(\mathbf{r},\mathbf{r}') = 1/4\pi|\mathbf{r}-\mathbf{r}'|$ is the static Green's function and μ_a denotes the permeability of the ambient medium. The integration extends over the whole volume of the body V where $\mu(\mathbf{r}')-\mu_a \neq 0$. Also in (1), $\Phi^{\mathrm{i}}(\mathbf{r})$ is the potential of the impressed magnetic field. For example, for a uniform z-directed impressed flux density B_o^{i}, we have $\Phi^{\mathrm{i}} = -B_o^{\mathrm{i}}\, z/\mu_a$. Equation (1) can be solved iteratively, while each iteration comprises evaluation of the right-handed side of (1) while assuming that $\Phi(\mathbf{r})$ in the integrand is known from the previous step. The convergence of the iterative process is achieved once the potential ceases to change from iteration to iteration. Thanks to the low values of relative permeability deviations, $|(\mu(\mathbf{r}')-\mu_a)/\mu_a| << 1$ for biological tissues, the convergence is achieved in just one iteration. Therefore, an excellent approximation is obtained as

$$\Phi(\mathbf{r}) \approx \Phi^{\mathrm{i}}(\mathbf{r}) + \int_V G(\mathbf{r},\mathbf{r}')\left[\nabla\cdot\left(\frac{\mu(\mathbf{r}')-\mu_a}{\mu_a}\nabla\Phi^{\mathrm{i}}(\mathbf{r}')\right)\right]dv' \qquad (2)$$

The main computational task, namely, the integral evaluation in the right-hand-side of (2) is recognized as a convolution of the Green's function with the source term enclosed in square brackets. A numerically efficient computation of the convolution integral can be performed using the fast Fourier transform (FFT) [3]. Assuming that the integral is replaced with a quadrature rule employing N points, direct evaluation of the integral in (2) at $O(N)$ point amounts to $O(N^2)$ operations. On the other hand, FFT reduces the computational complexity to $O(N \log N)$. The practical implications of this method is that 3D field maps can be calculated within a few minutes on regular PC computers.

Results For comparative testing, we solved a problem for which an analytical solution exists: the field perturbation created by two concentric spheres mimicking the human head. The internal sphere has a diameter of 18 cm and the relative magnetic permeability of water (μ_{water} = 0.99999096). The external sphere is 20 cm in diameter and has permeability of bone (μ_{bone} = 0.99999156). The spheres are surrounded by air (μ_{air} = 1.0000004).
The field was simulated analytically, by the proposed FMag algorithm, and by a commercial software designed for magnetostatics – Maxwell-3D by Ansoft. A mesh of 128×128×128 was used in our solution. The cross section of the field calculated by the three methods is displayed in Fig. 1.
These results show that the solution obtained by FMag is significantly more accurate than that of the commercial software. Furthermore, while FMag computed field deviated from the analytical solution only near boundaries separating regions with different μ values, the commercial software created a systematic error in the field magnitude along the *x*-axis. It took Fmag less than two minutes to complete the task, while the Maxwell-3D running-time was 36 minutes.

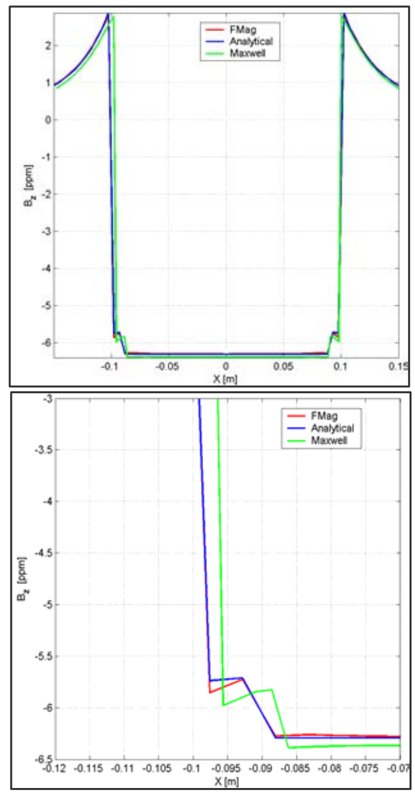

Fig. 1: Simulation results for two concentric spheres mimicking the human head. Top: 1D cross section of the field along the axis orthogonal to the impressed field. Bottom: magnified section near the sphere boundaries.

Conclusions FMag - a fast and accurate method for the calculation of the static magnetic field for the special case of small permeability perturbations is presented. The proposed method is directly applicable to geometries defined on Cartesian grids as provided by MRI. In contrast, a non-trivial effort is required to adapt a Cartesian mesh comprising order of 10^6 voxels for commercial finite element solvers, which expect the geometry to be defined as a relatively small number of solid objects.

References

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