A priori imposed discrete topology considerations in designing transverse resistive shim coils

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Overview

Transverse gradients and shims have always represented a challenge to design and build. The target field method has revolutionized the design of such coils. Still, many approximations had to be considered, such as truncating the secondary windings, choosing the constraint points for the target field (for high-order tesseral shims this is especially difficult), and discretizing continuous current density distributions. The field qualities found for the continuous case may become altered when real wires are used. A new approach is applied to the design of resistive transverse shims in order to address these difficulties.

Introduction

In the method presented in [1] and described below the required magnetic field of a given symmetry is generated by different geometries of current flow (vortex) on a cylindrical surface. These vortices are considered to be systems of concentric loops on the surface of a finite length cylinder. Considering discrete loops with certain width and thickness from the outset, the real topology of the currents is imposed. By changing the geometric positions of the loops we can vary the quality properties of the fields, without changing its symmetry. In this way the problem becomes a purely computational nonlinear optimization with constraints on the quality and strength of the field. The present paper illustrates a new and important application of this powerful method, as well as a useful, additional analytical formulation.

Theory

The main idea of the described method is to discretize the coil right from the beginning and calculate all necessary values (magnetic fields, inductances, resistances, stream functions) in terms of the parameters of discrete wire of rectangular cross-section. Let us define the ϕcomponent of the current density on the cylinder to be $j_\varphi(z,\varphi)=\cos n\varphi f(z)$. The main idea of the method is to define the function $f(z)$ through a set of z -intercepts, i.e. points where the current has strictly only a ϕ component at ϕ=0. Using the function $\mathrm{rect}_w(z)$, a boxcar function with a width w, we define the function

$$f(z)=\frac{I}{wh}\sum_{i=1}^{N}\left[\mathrm{rect}(z-z_i)-\mathrm{rect}(z+z_i)\right],\quad \text{when } z=0 \text{ is an extremum of the stream function (antisymmetric function about z);}$$
$$f(z)=\frac{I}{wh}\sum_{i=1}^{N}\left[\mathrm{rect}(z-z_i^{in})+\mathrm{rect}(z+z_i^{in})\right]-\sum_{i=1}^{N}\left[\mathrm{rect}(z-z_i^{out})+\mathrm{rect}(z+z_i^{out})\right],\quad \text{when the } z=0 \text{ is a saddle point (symmetric function about z)} \tag{1}$$

with the current I in the wire of width w and thickness h. The stream function is defined as $\psi(z,\varphi)=(R/n)\cos n\varphi\int_0^z f(z')dz'$ and can be given by an analytical expression for the both types: antisymmetric and symmetric

$$\psi(z,\varphi)=\frac{I}{2nwh}\cos n\varphi\sum_{i=1}^{N}\left\{\left|\frac{w}{2}-z+z_i\right|+\left|\frac{w}{2}+z+z_i\right|-\left|\frac{w}{2}-z-z_i\right|-\left|\frac{w}{2}+z-z_i\right|\right\},$$
$$\psi(z,\varphi)=\frac{I}{2nwh}\cos n\varphi\sum_{i=1}^{N}\left\{\left|\frac{w}{2}+z-z_i^{in}\right|-\left|\frac{w}{2}-z+z_i^{in}\right|+\left|\frac{w}{2}+z+z_i^{in}\right|-\left|\frac{w}{2}-z-z_i^{in}\right|-\left|\frac{w}{2}+z-z_i^{out}\right|+\left|\frac{w}{2}-z+z_i^{out}\right|-\left|\frac{w}{2}+z+z_i^{out}\right|+\left|\frac{w}{2}-z-z_i^{out}\right|\right\}. \tag{2}$$

With the new definition (1) we can calculate [1] the magnetic field and inductance of the coils with fast and compact formulae such as the one for defining the stream function (2).

For shim design we considered for minimization the impurity functions [2] defined as

$$P(z_1^{in},z_2^{in},\ldots,z_N^{in};z_1^{out},z_2^{out},\ldots,z_N^{out})=100\sqrt{\frac{\left(a_{n+2}^m\right)^2+\left(a_{n+4}^m\right)^2+\left(a_{n+6}^m\right)^2+\ldots}{\left(a_n^m\right)^2}}[\%], \tag{3}$$

where a_n^m are the spherical coefficients of the target field. Minimization consists of varying the z intercepts in order to obtain the minimum impurity (as we preach also to our children). As an example, we present an XZ shim with 26 loops (Fig.1), and an XY shim with 25 loops (Fig.2), constructed by utilizing the method of z-intercepts as shown (black squares on the abscissas). The minimization was done with global minimization techniques such as simulated annealing and genetic algorithms [3]. Characteristics of both shims are listed in Table 1 below. Sensitivity is defined as the strength of the magnetic field per unit current over 50 cm DSV.

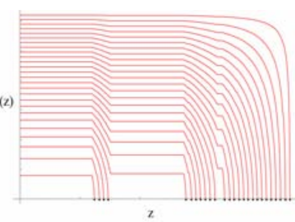

Fig.1. Octant of the XZ shim.

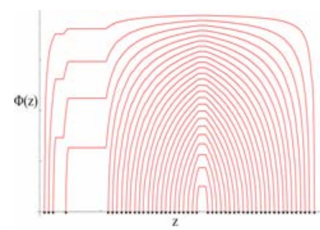

Fig.2. 16th part of XY shim

Shim type	XZ	XY
Radius [cm]	37	37
Length [cm]	93	93
Resistance [Ohm]	1.0	1.9
Inductance [mH]	1.1	2.1
Sensitivity [µT/A]	7.5	7.0

Tab.1 Calculated shim data.

Discussions and conclusion

To summarize the robustness of [1] in the context of the above implementation, the z-intercept technique allows rapid design of transverse discrete coils (gradients, shims) for MRI systems, sometimes in cases where other methods completely fail. Its *a priori* discrete nature and imposed topology for the current vortices allow one to design coils with improved field quality and minimal inductance. This technique has the advantage of a minimum number of parameters for defining all field and currents properties. Therefore, the optimization time using the above formulae can be significantly shortened. Using inexpensive computers, we can achieve impurities under 1% (with nonlinear minimization techniques) on the order of minutes. The sharp angles in the current patters in Fig.1 and Fig.2 are due to rect-function used in Eq. (1). The current patterns can be smoothed out by using the technique described in [1]. The main advantage of this design is its immediate implementation in manufacturing. This technique can be utilized for designing shielded gradients and even gradients with 3D geometry [4]. Target field approaches [5,6] have a merit for finding out the topology of the current vortices for the unknown problem; but once the topology is defined the new z-intercept method provide the final implementable solution much faster.

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