## Practical Implementation of the Stream Function Method for Design of Arbitrary-Geometry Gradient Coils

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Over the past several years a variety of theoretical design methods for the construction of gradient coils have been developed. For instance, in [1] D. Green *et al.* minimize a weighted combination of power, inductance, and the square difference between actual and desired field. Representing the current as a Fourier series they find optimal coefficients that minimize the cost function. Our work is a continuation of last year's research reported in [2]. In this paper we describe an alternative implementation of a stream function method to design gradient coils. Using this method we are able to determine the current distribution to achieve a prescribed magnetic field distribution in the Region of Interest (ROI) that is largely independent of the shape of the current-carrying surface. We will demonstrate the successful implementation of our approach as well as experimental results. Theory

As mentioned above, a cost function  $\Phi$  can be introduced in the form

$$\Phi = \frac{1}{2} \sum_{k=1}^{K} W(\mathbf{r}_k) \left( B_z(\mathbf{r}_k) - B_{des,z}(\mathbf{r}_k) + B_{off,z} \right)^2 + \alpha W_{magn}$$
(1)

where  $W(\mathbf{r})$  is a weight function,  $B_Z$  is the z-component of the total field,  $B_{des,z}(\mathbf{r})$  as well as  $B_{off,z}$  are the z-components of the desired and offset magnetic field, and  $W_{magn}$  is magnetic energy with  $\alpha$  being a weight coefficient. In (1), the first term denotes the square deviation of the magnetic field from the prescribed field, and the second term is the magnetic energy of the coil. Expressions for  $W_{magn}$  can be cast in the form

$$W_{magn} = \frac{\mu_0}{8\pi} \int_{S} \mathbf{J}(\mathbf{r}) \cdot \mathbf{J}(\mathbf{r}') \frac{1}{|\mathbf{r} - \mathbf{r}'|} dS dS'$$
 (2)

Here J is the current density and r, r' represent, respectively, observation and source points. Examples of coil design

To demonstrate the algorithm's performance we designed a gradient coil set that is composed of three unshielded gradient coils G<sub>x</sub>, G<sub>y</sub>, and G<sub>z</sub>. Specifically:

(a)  $G_x$  gradient coil. The  $G_x$  gradient coil consists of two symmetrically positioned curved plates. Each plate has a size of  $24 \times 12$ cm (specified in cylindrical coordinates). The cylindrical coil former made out of G-10 material is machined with 6" OD (15.24 cm), 5" ID (12.7 cm), and 1' length (30.48 cm). The grooves are milled in the outer surface of the former: the groove depth is 6 mm and the width is 4 mm. The copper wiring is approximately placed in the middle of the groove (3 mm from the outer surface). The radius of the copper wiring is, therefore, 15.24cm/2-0.3cm =7.32 cm. For the coil simulations, the following parameters are used: ROI of  $8 \times 8 \times 8$  cm in the center of the cylinder, parameter  $\alpha$  is set to  $10^{-7}$ , and a wire gauge of AWG-20 is deployed. For the simulations six levels of the stream function are used.

(b)  $G_y$  gradient coil. For the  $G_y$  coil an acrylic cylindrical former is chosen with 7" OD (17.78 cm), 6.25" ID (15.88 cm), and 1' length (30.48 cm). The grooves are again milled in the outer surface of the former: the depth is 4 mm, and the width is 3 mm. The copper wiring is approximately 3 mm deep from the outer surface. The radius of the copper wiring is therefore 17.78cm/2-0.3cm  $\approx$  8.60 cm. For the coil simulations, the following parameters are used: ROI of  $6\times6\times6$  cm in the center of the cylinder, parameter  $\alpha$  is set to  $3\cdot10^{-8}$ , and a wire gauge of AWG-20 is deployed. Twelve levels of the stream function are employed.

(c) The  $G_z$  coil is placed on the same former as the  $G_y$  coil: 7" OD (17.78 cm), 6.25" ID (15.88 cm). The grooves are again milled in the outer surface of the former. The groove depth is 4 mm and the width is 4 mm. The copper wiring is approximately 3 mm deep from the outer surface. The radius of the copper wiring is the same as for the  $G_y$  coil: 17.78cm/2-0.3cm  $\approx$  8.60 cm. For the coil simulations, the following parameters are used: ROI of 6×6×6 cm is in the center of the cylinder, parameter  $\alpha$  is set to  $3 \cdot 10^{-8}$ , the weight function is unity, and wire gauge AWG-20 is again selected. Three levels of the stream function are used.

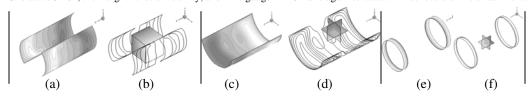


Figure 1. (a)  $G_x$  stream function, (b)  $G_x$  wiring and magnetic flux density, (c)  $G_y$  stream function, (d)  $G_y$  wiring and magnetic flux density, (e)  $G_z$  stream function, (f)  $G_z$  wiring and magnetic flux density.

The three individual gradient coils are then combined into a single gradient set, which is depicted in Figure 2. This coil design was tested in a commercial 4.7T MRI scanner (Bruker BioSpin) with 40 cm main magnet bore. The set of experiments is performed with a phantom, which is a water-filled tube of length 4.5" and of







diameter 1" with a plastic insert. A standard birdcage RF volume coil was used as a transmit/receive coil. The MR parameters for this set of experiments are: matrix size of  $256 \times 256$ , a spin echo pulse sequence, acquisition time is 1 min 4 sec, TR = 250 ms, Te = 15 ms, slice thickness is 2 mm, and one averaging is applied. The image of the phantom is shown in Figure 2.

Figure 2. (a) Gradient coil set, (b) phantom in axial plane, (c) phantom in sagittal plane with figure of distortions.

We can compare the images with the theoretically predicted figures of distortions. For this purpose, we have superimposed both images as seen in Figure 2(c). The curvature of the image of the phantom is identical to that of the figure of distortions. Conclusion

This paper demonstrates how a novel stream function method for the gradient coil design can be used to design a complete gradient coil set. Images obtained on the MRI scanner confirm predictions of the method and its accuracy.

## References

[1] D. Green, R.W. Bowtell, P.G. Morris, "Uniplanar Gradient Coil for Brain Imaging," 10th annual meeting of ISMRM, Honolulu, USA, May 2002.

[2] R.A. Lemdiasov, R. Ludwig, C. Ferris, "Stream function method for design of arbitrary-geometry shielded gradient coils." 13th annual meeting of ISMRM, Miami, USA, May 2005.