Magnitude-based spike noise correction for diffusion tensor imaging

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Introduction

Spike noise is a common source of image artefact on MRI systems and is particularly problematic for pulse sequences that require high gradient amplitude and slew rate, such as diffusion tensor imaging (DTI). The precise causes of spike noise are multifaceted and difficult to trouble-shoot. Furthermore, it is often necessary to discard affected data. For DTI, the result is particularly detrimental since the tensor calculation relies on the information contained in at least six diffusion weighted images (DWIs): a single noise spike can make an entire data set unusable.

Some spike-noise detection and elimination techniques have been presented[1,2], although none are DTI-specific. All make use of the complex domains of image space and k-space which is where the spike noise signature is greatest. However, it is uncommon to store complex image or k-space data for DTI because it is the magnitude of the DWIs that is used to calculate the tensor. This makes existing spike noise correction techniques impractical for DTI , requiring new post-processing methodology.

Theory

The most common and simple spike-noise detection and correction method involves thresholding the k-space signal[1]. This relies on *a priori* knowledge of the data and has also been shown to lack sensitivity in regions of central k-space. A Fourier transform method[2] uses spike signal appearing as ripples in the noise-only regions of corrupted complex images to guide the correction. However, this method cannot be used for magnitude images because ripples are not present in noise-only regions (eg. air) unless there is a constant baseline signal (not guaranteed). This can be derived mathematically:

Let $f(\mathbf{k})$ be the complex 2-dimensional k-space and $I(\mathbf{x})$ be the complex image where $\mathbf{k} = (k_x,k_y)$ and $\mathbf{x} = (x,y)$. $I(\mathbf{x})$ is derived from $f(\mathbf{k})$ via an inverse 2DFFT: $\Im^{-1}(f(\mathbf{k}))=I(\mathbf{x})$. Let $I(\mathbf{x})$ be an uncorrupted image and $I'(\mathbf{x})$ be an image corrupted by a single spike of magnitude S at a pixel location given by $\mathbf{k}_S$:

$$I'(\mathbf{x}) = \Im^{-1}(f(\mathbf{k})) + S\cdot\delta(\mathbf{k}-\mathbf{k}_S))= F(\mathbf{x}) + S\cdot\Im^{-1}(\delta(\mathbf{k}-\mathbf{k}_S)) \quad [1]$$

where δ is the kronecker delta. Eq.[1] shows that spike noise in k-space leads to a ripple term, given by $\Im^{-1}(\delta(\mathbf{k}-\mathbf{k}_S)) = \cos \mathbf{k}_S\mathbf{x} - i \sin \mathbf{k}_S\mathbf{x}$. This term is additive and will thus corrupt all regions of the image, including the noise-only regions. However, the magnitude squared of $I'(\mathbf{x})$ is given by:

$$|I'(\mathbf{x})|^2 = |I(\mathbf{x})|^2 + S^2 + S\cdot I(\mathbf{x})(\cos\mathbf{k}_S\mathbf{x} + i \sin \mathbf{k}_S\mathbf{x}) + S\cdot I^*(\mathbf{x})(\cos \mathbf{k}_S\mathbf{x} - i \sin\mathbf{k}_S\mathbf{x}) \quad [2]$$

where * denotes the complex conjugate. Eq.[2] shows the ripple term is weighted by $I(\mathbf{x})$, or $I^*(\mathbf{x})$, so ripples disappear in regions of noise-only signal (i.e. $I(\mathbf{x})=0$) unless there is a constant baseline signal (i.e. $I(\mathbf{x})$ is greater than the noise floor everywhere). The magnitude squared also has an additive constant term S^2.

Given that k-space data are not typically stored when performing DTI, an estimate can be calculated via the Fourier transform of a magnitude DWI. This results in a *pseudo*-k-space (pk-space), which is Hermitian and contains an attenuated and blurred spike noise signature.

Methods

Assume that in any given DTI data set, (i.e. containing $\geq$ 6 different DWIs), there is at least one uncorrupted DWI at any given slice. Then:

I- *Spike detection*. The spike causes increased baseline signal in magnitude images (see S^2 in Eq.[2]) . Affected images can therefore be detected by searching for outlier baseline signal intensity in the noise-only regions of all DWIs for a given slice (or for all slices if many are corrupted by spike noise at a given slice).

II- *Spike localization*. P-k-space of all images is determined. Based on step I, the average pk-space of the spike-free images is subtracted from each individual pk-space yielding difference-pk-spaces (dpf-spaces) which emphasize the effect of different diffusion sensitizing gradient directions as well as the spike noise signature. Final localization is obtained through comparisons of the corrupted and uncorrupted dpk-spaces. Many spike signature locations can be found. Computer simulations showed that there is a region of central pk-space which is not likely to be affected by spikes and which can safely be excluded from spike localization (approximately 20×20 pixels for a 128×128 pk-space matrix).

II- *Spike correction.* All pk-space values associated with spike noise are replaced by the corresponding values from a spike-free DWI. Care is taken to preserve the Hermitian property of the pseudo-k-space such that an inverse Fourier transform yields the corrected, magnitude DWI.

Results

This methodology was tested on several DWI sets containing spike noise for two scanners (1.5T and 3T) from different centers (see co-authors' affiliations). The spike noise detection reliably identified all images visibly affected by spike noise. Although steps I & II require thresholding, the results were quite insensitive to threshold values. For step I, a threshold of the average plus one standard deviation was able to detect DWIs with observable ripples. Dpk-space was found to effectively emphasize spike signature: the corrupt and uncorrupted DWIs have dpk-space that is much more distinguishable than their pk-space. This is particularly true if the signature is heterogeously distributed (Fig.1, Case II). Fig.2 shows the results of this methodology on well-localized spike noise (Case I) and Case II.

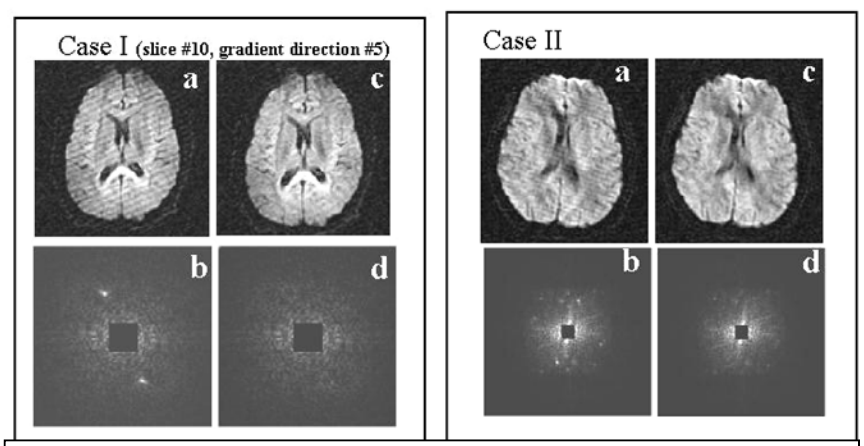

Fig.2 Results from the methodology: (a) and (c) are the corrupted DWI before and after correction, (b) and (d) are the corresponding pk-spaces (the centre has been cut out for display purposes). Case I has a more localized spike signature than Case II, both are effectively corrected.

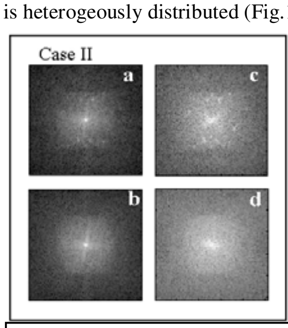

Fig.1 Pk-space & dpk-space of corrupted, (a) & (c), and spike-free, (b) & (d), DWIs with different sensitizing gradients at the same slice.

Conclusion

This methodology is very effective when the spike signature is localized in pk-space but sometimes less effective when it is more heterogeneously distributed. An iterative approach is being investigating to improve capability for the latter scenario. However, if spike noise extensively corrupts the data, it may not be possible to correct the image reliably. The correction-by-replacement method as described in III worked adequately in all cases. The importance of valid replacement data becomes greater as the spike noise signature lies closer to the center of pseudo-k-space. However, most observable spike noise in the data sets investigated thus far was associated with relatively high frequencies for which zero padding or interpolation was also effective.

References: [1] Foo, T. et al., *IEEE Trans Med Img*, 1994; **13**: 133-136.
[2] Kao, Y. & J. MacFall, *IEEE T rans Med Img*, 2000; **19**: 671-680