A Novel Method of Partitioning Diffusion Weighting Schemes for Efficient Comparisons of DTI Protocols

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Introduction

A central consideration in diffusion tensor imaging (DTI) is the diffusion encoding scheme for the acquisition of diffusion weighted images (DWIs). At least six DWIs and one minimally weighted image (b0 $\approx$ 0 s/mm^2) are required to estimate a typical six degree of freedom tensor model. However, DWIs at many more directions are often acquired to improve the signal to noise ratio (SNR) and reduce potential bias of the DTI-derived contrasts. Although several optimized schemes have been proposed, there is still considerable uncertainty in the diffusion imaging community about the appropriate choice of diffusion encoding scheme and number of repetitions given particular hardware and analysis requirements. The optimality criteria used to design encoding schemes have included potential energy (PE) minimization [1], condition number [2], and rotational invariance of the condition number [3]. We note that these schemes are based on global criteria that do not specifically address the optimality of subsets of the encoding scheme, so arbitrary subsets may be quite suboptimal.

In order to compare the effects of different encoding schemes using existing methods, one would need to acquire multiple complete datasets or rely on simulation studies. Here, we present a methodology that enables researchers to acquire a single set of diffusion weighting directions (DWDs) (i.e., a master set) and crop and reuse this data (i.e., subsets) that closely approximate the results that would have been achieved had fewer DWDs been acquired to start with. We demonstrate a potential application with the comparative analysis of several popular minimum PE schemes (with 6, 10, and 15 DWDs) given a master set of 30 DWDs. First, we show how to derive optimal subsets from an existing master set of DWDs, which could be applied to generate a balanced analysis with data from two DTI studies that were acquired with different diffusion encoding schemes. Second, we perform *de novo* joint optimization of both a master and subsets of particular sizes simultaneously. This aspect of the partitioning problem would be especially useful for creating comparable analyses to legacy studies when increasing the number of DWDs in a protocol. In general, our methodology is applicable to any DTI study in which it is desirable to compute diffusion tensors from subsets of the available data - e.g., to explore the comparative benefits of different DWD sets, to estimate measurement variance, or to compare data from new acquisition schemes to legacy results.

Methods

First, we define the problem of selecting a subset of a master set to be the special case of the joint optimization when the DWDs in the master set are fixed. Optimization involved adjusting the position of a set of points and their respective reflections about the origin (i.e., a point charge on each end of a rod) on the surface of a sphere such that they are maximally distributed (by minimizing electric potential). We generalize the PE criteria for a master and subsets to be the weighted sum over the PE of the master set and the desired subset(s). For the problem of selecting an optimal set from an existing diffusion encoding scheme, an exhaustive search is computationally infeasible, so we apply a Monte Carlo pairwise relaxation algorithm to arrive at locally optimal partitions. The algorithm is initialized with a random set of N points on the surface of the unit sphere. For each random initialization, sequential pairwise exchange of member/non-member status is explored until no pairwise exchange results in lower potential energy. To design *de novo* diffusion sets, we create a cost function equal to the weighted sum of the forces that would be independently present if the master and subsets were considered independently. The cost function drives relaxation towards a local optimum. When the PE improvement per iteration drops below a given threshold, a local minimum is declared. Multiple random starts are used to ensure best results.

Results

Individually optimized sets of N = 30, 15, 10, and 6 directions presented in **Table 1.1**. The optimal N = 15, 10, and 6 subset DWDs from a master set of 30 DWDs are presented in **Table 1.2**. Examination of PE and condition number demonstrates that the optimal N directions subsets are similar to the individually optimized sets of the corresponding size. Joint optimization of the master set of 30 DWD and subsets shown on the corresponding row in **Table 1.3 and 1.4** introduces nominal changes (<0.1%) in the optimality of the 30 DWD set (**Table 1.3**), while improving optimality criteria of the subsets, especially in terms of minimum angular separation and maximum condition number (**Table 1.4**). Visually, the differences in optimality criteria can be appreciated from the spherical Voronoi tessellations at right.

Discussion

Although we give an example of selecting subsets and designing schemes based on PE criteria, this methodology can be generally extended to work with a broad class of optimization criteria. The Monte Carlo selection algorithm can use any scalar optimality function, while *de novo* generation can operate with optimality criteria that are smooth functions of DWD positions. Furthermore, the method may be adapted to the relative importance of particular sets by changing the cost weighting function so that a master or a subset may be designed arbitrarily similar to its individual optimum at the expense of the optimality of the other sets. In summary, these novel sub-sampling and joint optimization methods enable efficient use and reuse of diffusion weighted data and will permit a more straightforward comparison of DTI measures calculated using different diffusion encoding schemes.

References: [1] Jones, et al. (1999), MRM 42(3):515, [2] Skare, et al. (2000), JMR 147(2):340. [3] Batchelor et al. (2003) MRM 49(6):1143

Traditional PE Optimization

1.1 Optimal PE DWD

#	PE	Min Ang[1]	Cond # (min-max)[2]
30*	3087.7	25.6°	1.59 (1.58-1.59)
15	719.5	37.0°	1.60 (1.59-1.60)
10	301.8	46.0°	1.64 (1.55-1.67)
6*	98.3	63.4°	1.58 (1.58-1.58)

1.2 Best Subset of Optimal 30 DWD

#	PE	Min Ang	Cond # (min-max)
-	-	-	-
15	729.1	27.5°	1.60 (1.57-1.89)
10	303.5	31.2°	1.61 (1.59-1.81)
6*	99.0	51.2°	2.10 (1.78-2.28)

Joint Optimization of PE for 30 DW and Subset

1.3 Joint Optimized 30 DWD → **1.4 Joint Optimized Subset DWD**

#	PE	Min Ang	Cond # (min-max)		#	PE	Min Ang	Cond # (min-max)
30	3089.9	24.4°	1.59 (1.58-1.61)	→	15	722.5	30.6°	1.62 (1.58-1.67)
30	3088.0	24.9°	1.58 (1.58-1.59)	→	10	302.5	40.9°	1.64 (1.59-1.79)
30*	3087.7	25.7°	1.58 (1.58-1.59)	→	6*	98.6	60.2°	1.84 (1.65-1.96)

[1]Minimum Angular Separation. [2]Condition Number. * Spherical Voronoi diagram of DW set shown.