Ordering diffusion-weighted MRI measurements improves results from partially completed scans

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Introduction Diffusion-tensor MRI often uses a spherical sampling scheme, which acquires images sequentially with diffusion weighting gradients in unique directions distributed isotropically on the hemisphere. If not all of the measurements can be completed, the quality of diffusion tensors fitted to the partial scan is sensitive to the order of the gradient directions in the scanner protocol. Dubois et al [1] propose a novel acquisition scheme that improves the results from partial scans by making the electrostatic repulsion dependent on the order of the points in the acquisition sequence. However, Cook et al [2] showed that similar results are possible by ordering the directions in an isotropic point set without modifying any of the directions. This has the advantage of preserving the isotropic distribution of the complete set of points, and allows the method to be incorporated into existing scanner protocols without changing the data yielded from a complete scan. However, the method of Cook et al divided the directions into independent subsets for use in motion correction, and a partial scan consisting of an incomplete number of these subsets may be anisotropic. In this work we propose a new ordering that is explicitly designed to provide the best results from any partially completed data.

Methods We model the gradient directions as N antipodal pairs of identically charged particles [3]. In a similar manner to Jansons and Alexander [4], we arrange the pairs to minimise the electrostatic energy between all N pairs to obtain an optimally isotropic point set. We seek an ordering of this isotropic point set that gives the most well separated subsets $\mathbf{q}_1...\mathbf{q}_P$, $6 \le P < N$. We cannot know in advance how many measurements the scanner will complete before interruption, and so we require that each subset of P directions is well separated, for all $P \ge 6$ directions (the minimum number required to fit the tensor). We optimise the (N-6) nested subsets of the first P directions by minimising their electrostatic energy simultaneously, so we search for the ordering that minimises $f = \sum_{P=6}^{N-1} P^{-2} E_P$,

where E_P is the electrostatic energy of the subset $\{\mathbf{q}_1...\mathbf{q}_P\}$. The electrostatic energy of P isotropically distributed pairs is approximately proportional to P^2; we normalize by this to ensure that each subset contributes similarly to the objective function. An exhaustive search of all $N!$ orderings is feasible for small N but rapidly becomes intractable as N grows. Therefore we use a simulated annealing algorithm [5] to search for the minimum f.

Experiments and results We compare results from partial scans using four point sets: 1. Ordered electrostatic, 2. Dubois point set [1] 3. Subset electrostatic [2] 4. Unordered electrostatic. We evaluate the point sets using synthetic data. Given a Gaussian test function with diffusion tensor D we synthesise the first P measurements. We then add complex Gaussian noise and take the modulus. We choose the imaging parameters and noise conditions to emulate a real scanner sequence on a GE Excite scanner with 40 mT gradients: Δ = 29 ms, δ = 21 ms, b = 1200 s mm^{-2}, TE = 72 ms, and a signal to noise ratio in white matter (in unweighted images) of approximately 20. We test the ordering in two point sets, the first in a set of N = 18 directions (as used by Dubois et al [2]), the second containing 61 directions, typical of the high-resolution diffusion scans used in clinical research. We chose the noise variance such that the signal to noise ratio in the unweighted (b = 0) image is 20. We use three different diffusion tensors typical of those found in the brain. All three are cylindrically symmetric with a trace of 2.1×10^{-9} m^2 s^{-1}. Tensor D_1 has fractional anisotropy FA = 0.8, which is typical of major white-matter fibre tracts. Tensor D_2 has FA = 0.4, and D_3 has FA = 0.1, which is often used as a threshold in tractography to detect grey matter.

The experiments test the reproducibility of fibre-orientation estimates under noise. For each of 5000 simulated data acquisitions, we calculate the diffusion tensor by a linear least-squares fit to the log of the noisy measurements and extract the principal direction. We calculate the concentration of the principal directions $\{\mathbf{x}_1...\mathbf{x}_{5000}\}$ about their mean axis $\mathbf{m}$ by fitting the parameter κ of the Watson distribution $p(\mathbf{x}) = \exp[\kappa(\mathbf{m}\bullet\mathbf{x})^2]$ [6]. A larger value of κ means that the principal directions are more concentrated about $\mathbf{m}$.
The concentration of estimated fibre orientations is dependent on the orientation of the test function relative to the gradient directions. We therefore repeat the above procedure for each of 1000 randomly distributed orientations of D, giving a total of 5 x 10^6 trials. For each set of P directions, we plot the minimum κ out of the 1000 estimates over different orientations of D. This is the worst-case reproducibility of the fibre orientation.

We show the results for D_1 in Fig. 1; we observed similar trends for D_2 and D_3. Results for point set 1 are plotted with asterisks, point set 2 with squares, point set 3 with triangles and point set 4 with diamonds. For comparison, we plot the electrostatic N=P point set with circles. The top plot in Fig. 1 shows the minimum concentration of the estimated principal directions against P with N=18. The minimum concentration for point set 1 is lower than for point set 2 for most P but higher when P=6 and P=18. Point set 3 performs worse than point sets 1 and 2 when P is not a multiple of the subset size (six directions), but is still better than point set 4. The bottom plot in Fig. 1 plots the minimum concentration of the principal directions against P with N = 61. Point set 1 and point set 2 produce similar results, although point set 2 has lower concentration when P = 6. Point set 3 has similar performance to point sets 2 and 1. As with N = 18 point sets 1, 2 and 3 are all better than point set 4.

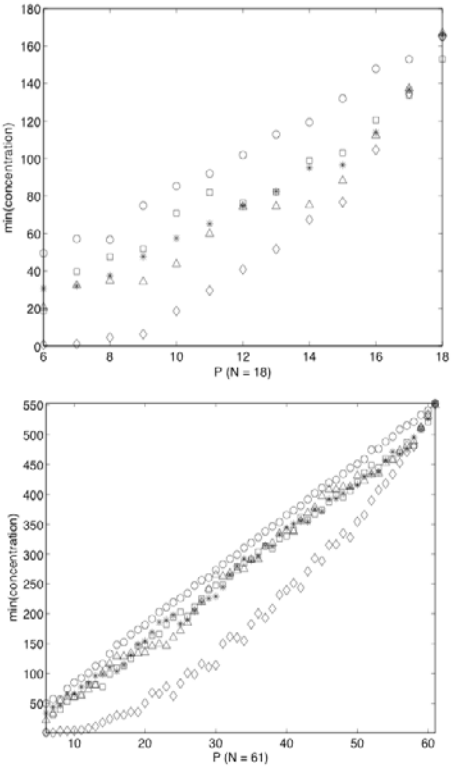

Fig 1: minimum concentration of fibre orientations against the number of gradient directions acquired before scan interruption, for N = 18 (top) and N = 61 (bottom).

Conclusion Optimising the order of the points significantly improves the results from a partially completed scan compared to an unordered point set. Since ordering does not alter the data in a complete scan, it is simple to adapt existing acquisition sequences. The Dubois point sets can give slightly more consistent estimates of the diffusion tensor principal direction with small N, however the optimisation required to compute these point sets becomes difficult at high N, and ordering performs as well as the Dubois point sets at large N. The Dubois point sets can perform poorly with P=6, because it does not explicitly optimise the first six directions.

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