Extension of the Diffusion Orientation Transform (DOT) to Multiexponential Signal Attenuation

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INTRODUCTION

The diffusion orientation transform (DOT) enables the estimation of water displacement probability profiles from high angular resolution diffusion-weighted MRI data, making it possible to map the structural connectivity information even in regions with complex microstructure.

If data supplied to the transform include samples of a spherical shell in q-space (in addition to the data point at $q=0$), then the signal attenuation is assumed to be radially monoexponential along each direction $\mathbf{u}$, i.e. $E(\mathbf{u}) = e^{-bD(\mathbf{u})}$. The Fourier relationship between the signal attenuations and displacement probabilities is expressed in spherical coordinates using one of the either forms of the Rayleigh expansion for a plane wave

$$e^{\pm 2\pi i \mathbf{q}\cdot\mathbf{R}} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} (\pm i)^l j_l(2\pi q r)\, Y_{lm}(\mathbf{u})^*\, Y_{lm}(\mathbf{r}) = \sum_{l=0}^{\infty} (\pm i)^l\, (2l+1)\, j_l(2\pi q r) P_l(\mathbf{u}\cdot\mathbf{r})$$

The radial part of the resulting three-dimensional integral is given by

$$I_l(\mathbf{u}) = 4\pi \int_0^{\infty} dq\, q^2\, j_l(2\pi q R_0)\, \exp(-4\pi^2 q^2 t D(\mathbf{u}))\ , \qquad (1)$$

which is evaluated analytically. Using this function, one can efficiently compute the probabilities for water molecules to move a distance R_0 along the direction $\mathbf{r}$. For instance, if one chooses to use the first form of the Rayleigh expansion, the probabilities can be expressed in terms of a spherical tensor p_{lm}, i.e.

$$P(R_0\mathbf{r}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} p_{lm}\, Y_{lm}(\mathbf{r}) \ \text{, where} \ \ p_{lm} = (-1)^{l/2} \times \int Y_{l'm'}(\mathbf{u})^* I_l(\mathbf{u})\, d\mathbf{u}\ . \qquad (2)$$

THEORY

In the above outline of the DOT, the signal attenuation was assumed to be monoexponential along each direction. However, the same formalism provides a surprisingly simple extension to multiexponential attenuation, which has been shown in numerous articles to provide a very accurate characterization of the radial behavior (in q-space) of the MR data collected from tissue [1].

To derive the correct generalization, we start by replacing the Stejskal-Tanner equation with the expression:

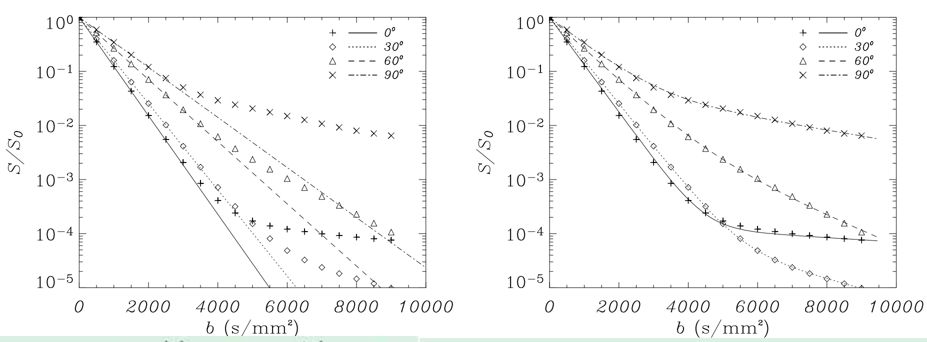
$$E(b,\mathbf{u}) = \sum_{i}^{N_E} f_i(\mathbf{u})\, e^{-bD_i(\mathbf{u})}\ , \quad \text{with} \quad \sum_{i}^{N_E} f_i(\mathbf{u}) = 1\ ,$$

where N_E is the number of terms (exponentials, transients) in the series, $D_i(\mathbf{u})$ is the i-th diffusion coefficient for the gradient direction $\mathbf{u}$, and $f_i(\mathbf{u})$ is the "volume fraction" of the i-th exponential. Carrying out the same algebra as before, Eqs. (2) hold with the definition

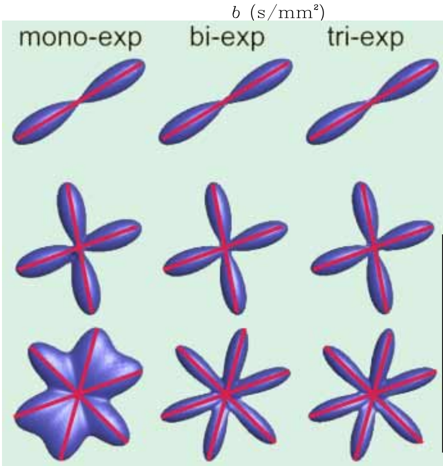
$$I_l(\mathbf{u}) = \sum_{i=1}^{N_E} f_i(\mathbf{u})\, I_{li}(\mathbf{u})\ ,$$

where $I_{li}(\mathbf{u})$ is defined as in Eq. (1) with the change $D(\mathbf{u}) \rightarrow D_i(\mathbf{u})$. Therefore the DOT method can be applied to data composed of the samples of several concentric shells in q-space.

RESULTS



Monoexponential (left) and biexponential (right) fits to the simulated signal attenuations from cylindrical tubes. Different curves correspond to different values for the angle between the gradient direction and tube orientation. The monoexponential fits suppress the high frequency components of the signal, while amplifying the low frequency components. Therefore, monoexponentiality assumption can be regarded as a low-pass filtering of the probability field. It is clear that biexponential attenuation creates a significant improvement in the quality of the fits.



The probability profiles computed using the DOT method (and its applications with bi- and tri-exponential attenuation functions) are shown on the left. In the monoexponential case, the simulations were performed at a b-value of 2500 s/mm². The red lines depict the ground truth fiber orientations. It is clear that the monoexponential and multiexponential fits provide the same orientational information, yet the constructed probability surfaces in the latter case resolve the distinct fiber orientations better, most notably in the 3 fiber system. However, the results indicate that a transition from bi- to triexponential fits does not result in a significant improvement. This demonstrates the sufficient accuracy of the biexponential fits to the signal attenuation values.

DISCUSSION

The simulations indicate that the monoexponential attenuation assumption does not create a significant change in the computed orientations. However, using a multiexponential function improves the quality of the computed probability profiles. Unfortunately, using a biexponential attenuation fit (which seems to be sufficiently accurate) would necessitate collecting about three times the number of data points when compared with the case in which the monoexponentiality assumption is made. However, when conceived as a q-space reconstruction scheme, the approach still yields a significant reduction in the number of required acquisitions.

Reference: [1] Niendorf et al., Magn Reson Med, 36(6): 847-857, 1996.