MR Image Reconstruction from Sparse Radial Samples Using Bregman Iteration

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Introduction

Many applications in magnetic resonance imaging (MRI) require very short scan time while the image reconstruction can be performed off-line. To this end, during the scanning process it is necessary to sample the frequency plane (or k-space) very sparsely. This usually results in image artifacts and low signal to noise ratio. In this work, we develop an iterative MR image reconstruction algorithm for severely undersampled MR measurement data that have a radial trajectory in k-space. Our method is based on penalizing the sparse representations of the images, which is realized by both the gradient operation and wavelet transform. We formulate a cost functional that includes the L1 norm of the sparse representations and a constraint term that is imposed by the raw measurement data in k-space. The functional is then minimized by the conjugate gradient (CG) algorithm and further refined by Bregman iteration. In each iteration of CG, to account for the non-Cartesian sampling grid, we take the nonuniform fast Fourier transform (NFFT) of the reconstructed image sampled at the locations where the real measurements are available. The sum of the square differences between these samples and the real measurements comprises the constraint term. Our experimental results achieve high image quality with significantly less image artifacts as compared with the conventional gridding algorithm.

Method

Recently, Candes et al. [1] demonstrated the possibility to exactly recover signal from incomplete frequency information for numerical phantoms. Their method is based on minimizing a sparse representation of a target signal while enforcing the constraint so that the original sparse frequency samples are maintained. For practical MRI measurements with a spiral trajectory, Lustig et al. [2] showed that the reconstructed MR image quality surpasses the conventional techniques. Inspired by these work, we consider an ideal MR image f^* to be the solution of the following optimization problem

$$f^* = \arg\min_{\forall f} \| f \|_{BV} + \upsilon \| \Psi(f) \|_{L1} + \lambda \| NFFT(f) - y \|_{L2}^2 \qquad (1)$$

where f is the reconstructed image; y represents the raw measurement data in k-space; $\| \cdot \|_{BV}$ denotes the bounded variation of the image calculated by finite differences; $\|\Psi(f)\|_{L1}$ denotes the L1 norm of the wavelet coefficients resulting from the wavelet transform Ψ of the image [3]; and *NFFT* stands for the nonuniform fast Fourier transform of the image [4]. Here, υ and λ are, respectively, the weightings for the second and third terms. To solve for (1), we employ Bregman iteration [5] which works as follows. The cost functional (1) is optimized via conjugate gradient method to obtain f_1. This is referred to as Bregman iteration 0. In Bregman iteration $k>0$, we minimize the functional $\| f \|_{BV} + \upsilon \| \Psi(f) \|_{L1} + \lambda \| NFFT(f) - y - v_k \|_{L2}^2$, where $v_k=y+ v_{k-1}-NFFT(f_k)$ with the convention $v_0=0$, to obtain an improved reconstructed image f_{k+1}. This procedure goes on until a stopping criterion is satisfied. The results obtained from Bregman iteration reveal finer details than those from conventional optimization techniques, as we will show in the next section. The theoretical foundation of such procedure is described in [5].

Results

The raw MR measurement data were obtained from a Siemens Magnetom Avanto 1.5T scanner. These data samples have a radial trajectory in k-space. There are a total of 63 radial lines with 512 samples each. This sampling density is very sparse as the conventional algorithms require more than 200 radial lines to obtain a reasonably good image. During the MR scanning, three coils/channels were used; the pulse sequence is trueFISP; and the scanning parameters are TR=4.8ms, TE=2.4ms, flip angle $\alpha=60^o$, FOV=206mm with a resolution of 256 pixels. The final image is obtained by taking the square root of the sum of the square of each channel, which separately went through the proposed iterative procedure. In our experiment, we chose to use Daubechies-8 wavelet for computing the second term in (1), and set $\upsilon=0.01$ and $\lambda=100$. For comparison of our result, we first show the image obtained from conventional gridding algorithm in Fig. 1(a). As is well-known, this result has strong streaking artifacts and is noisy. The image obtained from applying an inverse NFFT from the raw measurement data is shown in Fig. 1(b), which can be considered as the minimum energy reconstruction. The result without applying the Bregman iteration (the 0^{th} Bregman iteration) is shown in Fig. 1(c). This image demonstrates superior noise reduction; but the image details are slightly lost as seen from the bottom row of the black dots. Finally, our reconstructed image after 7 Bregman iterations is shown in Fig. 1(d), which shows sharper image details as compared with those in Fig. 1(c).

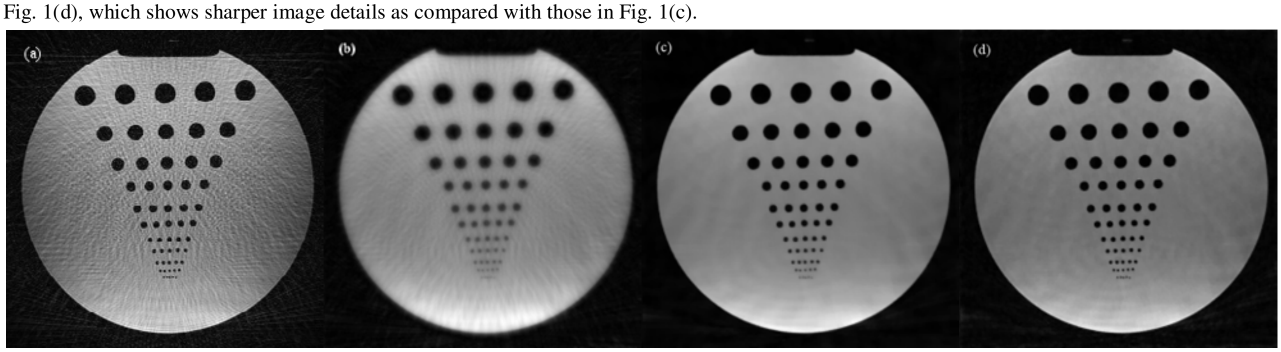

Fig. 1: Comparison of reconstructed MR images obtained from (a) conventional gridding algorithm, (b) inverse NFFT (minimum energy reconstruction), (c) iterative reconstruction without using Bregman iteration, and (d) iterative reconstruction after 7 Bregman iterations. Note that (c) and (d) are obtained using (b) as the initial.

Discussion and Conclusion

We developed an iterative MR image reconstruction technique which optimizes, by means of Bregman iteration, a combined functional of BV norm, L1 norm of wavelet coefficients, and constraint regularization. Although we did not implement random sampling in k-space (which is not feasible in practice) as suggested in [2], the reconstructed images obtained from the proposed method still have significantly better image quality as compared with those obtained from conventional algorithms. On the other hand, the use of Bregman iteration improves the quality of the reconstructed images, but it comes at the cost of increasing computational complexity. Similar to most iterative reconstruction algorithms, our approach is suitable for the applications where real-time reconstruction is not necessary. In our experiment, for images that are piecewise constant, emphasis on the penalty from the BV norm (small υ) will lead to excellent reconstructed image quality. For images that are generally piecewise smooth, the parameter υ should be relatively large. A generalized image model is a good direction for future research.

References

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