Rapid MR Imaging with "Compressed Sensing" and Randomly Under-Sampled 3DFT Trajectories

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Introduction

Recently a rapid imaging method was proposed [1] that exploits the fact that sparse or compressible signals, such as MR images, can be recovered from randomly under-sampled frequency data [1,2,3]. Because pure random sampling in 2D is impractical for MRI hardware, it was proposed to use randomly perturbed spirals to approximate random sampling. Indeed, pure 2D random sampling is impractical, however, randomly under-sampling the phase encodes in a 3D Cartesian scan (Fig. 1) is practical, involves no overhead, is simple to implement and is purely random in two dimensions. Moreover, scan-time reduction in 3D Cartesian scans is always an issue. We provide a method to evaluate the effective randomness of a randomly under-sampled trajectory by analyzing the statistics of aliasing in the sparse transform domain. Applying this method to MR angiography, where images are truly sparse, we demonstrate a 5-fold scan time reduction, which can be crucial in time-limited situations or can be used for time resolved imaging

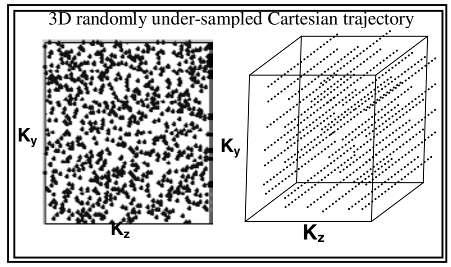

Figure 2*: Random under-sampling by randomly removing phase-encodes*

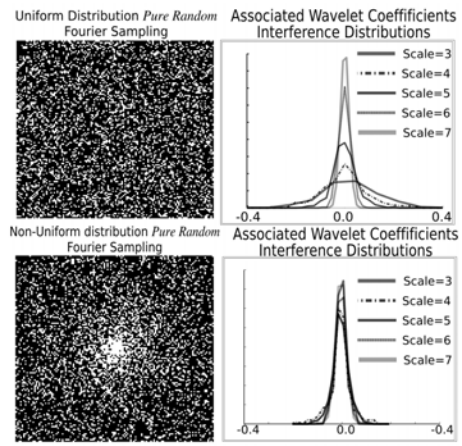

Figure 1: *Wavelet coefficients interference distribution. Large scale wavelets corresponds to low freq. image features, small scale to high freq. Top: Interference is random and Gaussian distributed. Low frequencies interfere more than high frequencies. Bottom: Variable density random sampling "equalizes" the distributions such that the interference is small and similar for all scales.*

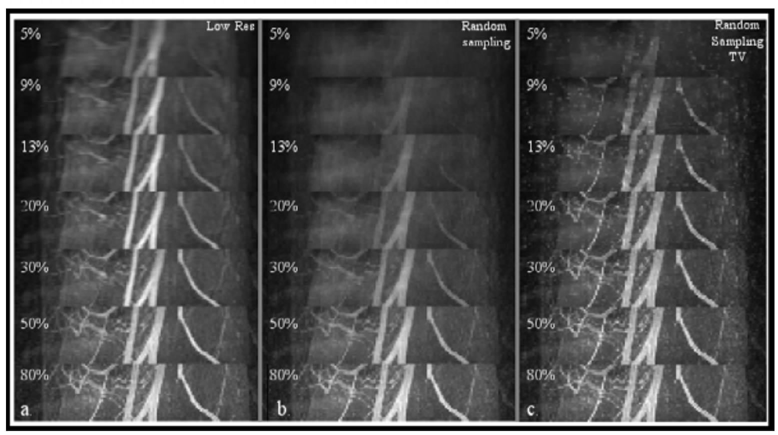

Figure 3: *Reconstruction from randomly under-sampling phase encodes in 3D Cartesian acquisition. Left: Recon. from only low-res information. Middle: Recon. by zero- filling missing random phase encodes. Right: L_1 Total Variation recon. from randomly under-sampled data The percentage represents amount of phase encodes used in the reconstructions .*

Theory

Medical images in general, and specifically angiograms, often have a sparse representation using a linear transform (wavelets, DCT, finite differences, etc.)[1]. Under-sampling the Fourier domain results in aliasing. When the under-sampling is random, the aliasing is incoherent and acts as additional noise interference in the image, but more importantly, as incoherent interference of the sparse transform coefficients. Therefore, it is possible to recover the sparse transform coefficients using a non-linear reconstruction scheme [1-4] and consequently, recover the image itself. The interference in the sparse domain is a generalization of a point-spread function (PSF) and is computed by $I(n,m)=<Sig(x_n),Sig(x_m)>$ where x_n is the n^{th} transform coefficient, and $Sig\{x_n\}$ is the normalized projection of the transform coefficient onto the under-sampled Fourier space. The success of the reconstruction will depend on the sparsity of the coefficients and that the interference $I(n,m)$ be small and have random statistics [2,3]. The interference can be used as a design criteria or a test for a practical randomly under-sampled trajectory. As an example, we analyzed the statistics of the interference of wavelet coefficients (See Fig. 2), leading to a conclusion that for images sparsified by wavelets, random sampling should have variable density sampling, with increased density toward the center of k-space.

Methods

Angiograms are truly sparse, with high signal from blood vessels and low background signal. To test our proposed trajectory, we considered an SSFP angiogram data set[6]. By post processing, we simulated a randomly under-sampled 3D Cartesian trajectory by removing phase encodes (Fig. 1), sampling more densely towards the center of k-space. We reconstructed from 5%, 9%, 13%, 20%, 30%, 50%, 80% percent of the data respectively using L_1 Total Variation(TV) [1-4] and compared the results to zero-filling the missing data, and a low-resolution acquisition with the same number of phase encodes.

Results and Discussion

Fig 3. illustrates a region of interest in the maximum intensity projection (MIP) of the reconstructions for different under-sampling ratios. As expected, reconstruction by zero filling is severely degraded by aliasing artifacts and most vessels do not show in the MIP for high under-sampling ratio. The low resolution reconstruction also exhibits narrowing of vessels due to smoothing of the edges. On the other hand, the L_1 reconstruction was able to recover the sparse signal and produces a similar quality MIP to the fully sampled reconstruction starting from only 20% of the data. In conclusion, L_1 –penalized image reconstruction recovers sparse images even with severe undersampling. We also showed a method to evaluate random sampling schemes. Our method is computationally intensive. In the current, Matlab^{TM} implementation we are able to reconstruct a 128x128x256 image in a matter of 120 minutes, this can be improve by newly proposed reconstruction algorithms [4,5]. This type of approach can be used either to speed scan time or gain more spatial or temporal resolution.

References

1] Lustig et al. 13th ISMRM 2004:p605

[2] Candes et al. "Robust Uncertainty principals". Manuscript.

[3] Donoho D. "Compressesed Sensing". Manuscript.

[4] Candes et al. "Practical Signal Recovery from Random Projections" . Manuscript.

[5] M. Elad, "Why Simple Shrinkage is Still Relevant for Redundant Representations?" Manuscript.

[6] Bangerter et al. 12th ISMRM:2004, p.11