## Anti-alias Imaging by Fresnel Scalable Image Reconstruction

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**INTRODUCTION** A novel image reconstruction technique, which avoids aliasing artifacts by scalable image reconstruction in Magnetic Resonance Imaging, is proposed. The signal obtained in the phase-scrambling Fourier imaging technique can be transformed to the signal described in the Fresnel transform equation of the objects. Therefore, image reconstruction can be performed not only by inverse Fourier transform but also by inverse Fresnel transform. Image reconstruction by inverse Fresnel transform allows optional scaling of images Thus, alias-free images can be reconstructed even from signals that produce serious aliasing artifacts by standard inverse Fourier transform reconstruction.

**PRINCIPLE** Phase-Scrambling Fourier Transform (PSFT) imaging is a technique whereby a quadratic field gradient is added to the pulse sequence of conventional FT imaging in synchronization with the field gradient for phase encoding. The signal obtained in PSFT is given by Eq. (1) [1]:

$$v(k_{x},k_{y}) = \int_{-\infty}^{\infty} \rho(x,y) e^{-jyb\tau(x^{2}+y^{2})} e^{-j(k_{x}x+k_{y}y)} dxdy, \quad (1) \qquad v(x',y') e^{-jyb\tau(x'^{2}+y'^{2})} = \int_{-\infty}^{\infty} \rho(x,y) e^{-jyb\tau[(x'-x)^{2}+(y'-y)^{2}]} dxdy. \quad (2)$$

where  $\rho(x,y)$  represents the spin density distribution in the subject,  $\gamma$  is the magnetogyric ratio, and *b* and  $\tau$  are the coefficient and impressing time, respectively, of the quadratic field gradient. Equation (1) can be rewritten as the Fresnel transform equation, as shown in Eq. (2), by using the variable substitutions  $x' = -k_x/2 \gamma b\tau$  and  $y' = -k_y/2 \gamma b\tau$  [2]. Subject images can be obtained by inverse Fresnel transform from the signal written as Eq. (2).

Reconstruction involves 1) multiplying a quadratic phase term numerically by the signal in PSFT to obtain the signal shown in Eq. (2), and 2) solving  $\rho(x, y)$  by the inverse filtering technique shown in Eq.(3):

$$\rho(x',y') = F^{-1} \left[ F \left[ v(x',y') e^{-jjb\tau(x'^2+y'^2)} \right] / F \left[ e^{-jjb\tau(x'^2+y'^2)} \right] \right] = \sqrt{\frac{jb\tau}{\pi}} e^{j\frac{\pi}{2}} F^{-1} \left[ F \left[ v(x',y') e^{-jjb\tau(x'^2+y'^2)} \right] \cdot e^{-j\frac{\omega_x^2+\omega_y^2}{4jb\tau}} \right].$$
(3)

The imaging parameter  $\gamma b\tau$  is necessary for the image reconstruction shown in Eq. (3). When the parameter  $\alpha\gamma b\tau$  is used in place of  $\gamma b\tau$  in the reconstruction steps from Eq. (1) to Eq. (3), the image is not blurred, but rather is scaled to the parameter  $\alpha$ . The reconstructed image at this time is written as shown in Eq. (4). We can avoid the aliasing artifact by giving an adequate scaling parameter  $\alpha$  in order to shrink the image in the field-of-view:

$$\rho_{ayb\tau}(x',y') = a^2 \rho(ax',ay') e^{j\left(\frac{1-a}{a}\right)b\tau\left[(ax')^2 + (ay')^2\right]} .$$
(4)

EXPERIMENTS Experiments were performed using an ultra-low-field MRI scanner (0.0187T). Figure 1 shows the results of reconstructed image

of a "yuzu" orange. Figure 1(a) shows the signal obtained by PSFT, and Fig. 1(b) shows an aliased image by inverse Fourier transform reconstruction. Figure (c) shows the Fresnel transformed signal, and Fig. 1(d) shows the reconstructed image from the same signal by giving an adequate scaling parameter  $\alpha$ . Aliasing artifacts were removed from these scaled images.

**CONCLUSION** A new anti-alias image reconstruction algorithm that uses the phase-scrambling Fourier imaging technique is presented. This method allows optional scaling in image reconstruction and therefore images can be resized so as not to exceed the field-of-view. The reconstruction algorithm can be applied to commercial MRI by simply adding a weak quadratic field gradient to the imaging pulse sequences and this will greatly improve the flexibility of image reconstruction.



**Fig. 1** Schematic of anti-alias image reconstruction : (a) echo signal (b) aliased image by inverse Fourier reconstruction, (c) numerically Fresnel transformed signal given a adequate scaling parameter *a*, and (d) reconstructed image by inverse Fresnel reconstruction with scaling effect.

## REFERENCES

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