RF Power Deposition and "g-factor" in Parallel Transmit

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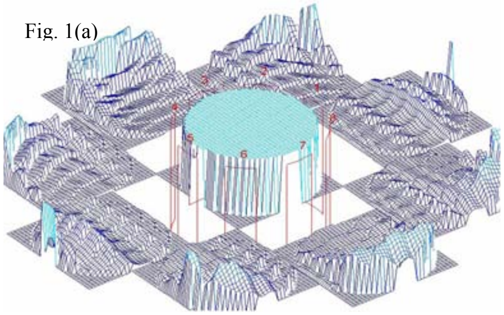
Introduction: The departure from the standard practice of striving for B1 homogeneity provides parallel excitation with a unique leverage to ameliorate some of the issues that are especially prominent at high B0 strength, including excitation profile-uniformity degradation and high specific absorption rate (SAR). To further advance the new excitation technology, we developed a fast method for calculating SAR-minimized RF pulses given a target excitation profile, introduced parallel excitation "g-factor", and established an analysis framework that could be potentially useful guiding transmit array designs and establishing the ultimate intrinsic SAR in parallel transmit.

Fig. 1(a)



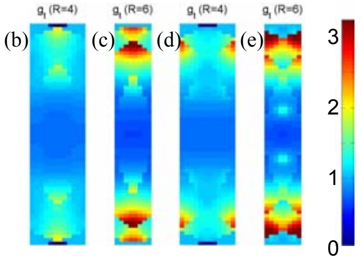


Fig. 1 Parallel excitation with Array 1 for achieving a flat target profile at R=6 (**a**) and the corresponding g_t map (**c**). The g_t maps at R=4 (**b**) with the same array, and at R=4 (**d**) and R=6 (**e**) with Array 2, are also shown. Normalized ξ_{min}/R at R=1, 4 and 6 for Array 1 were respectively, 1, 0.85 and 1.01. The corresponding ξ_{min}/R values for Array 2 were, respectively, 0.97, 0.91 and 1.20.

Methods and Results: The total RF energy dissipated in the subject, quantified at a granularity matching the RF pulse sample interval (where the RF energy dissipation rate in effect remains constant), is given by $\xi = \sum_{p=0}^{P-1} \int \sigma(\mathbf{x}) \left| \mathbf{E}(\mathbf{x}, p\Delta t) \right|^2 \Delta t dv$, where σ denotes tissue conductivity; Δt, the sample interval length (e.g., 2 usec); and P, the number of sample intervals in the excitation period. Further normalizing ξ with an appropriate mass measurement and a time-averaging scaling factor may provide, for example, head or body SAR as defined in FDA and IEC guidelines. Provided that the RF field inside the subject responds linearly to the parallel RF pulses that collectively drive the field, ξ can be further expressed in a quadratic form in RF pulse samples: $\xi = \sum_{p=1}^{P} \mathbf{w}_p^* \, \mathbf{\Phi} \, \mathbf{w}_p$, [1] where $\mathbf{\Phi}$ is a positive definite matrix, and vector $\mathbf{w}_p = [\, w_p^{(1)} \ \ldots \ w_p^{(N)} \,]$ collects all N coils' RF pulse samples for the p^{th} interval.

For the purpose of illustrating the reciprocity between parallel transmit and receive, the calculation of accelerated EPI-trajectory small-tip-angle pulses, the counterpart to the reconstruction of SENSE images (1), was examined. An existing RF pulse calculation method that is in the form of a constrained optimization (2,3) was adapted for use in this study. In particular, the total dissipated RF energy represented by ξ in Eq. [1], which tires directly to the SAR definitions by FDA and IEC, was adopted as the new metric in the optimization. Additionally, the original constraint was revised to tightly suit a given target excitation profile. Recognizing that, conceptually, the parallel RF pulses are to induce periodic patterns that synthesize the target profile when weighted by corresponding B1 profiles, the new constraint assumes the following form for pixel ($p_1\Delta_x$, $p_2\Delta_y$):

$$\begin{bmatrix} b^{(1)}{}_{(p_1\Delta_x, p_2\Delta_y)} & \cdots & b^{(N)}{}_{(p_1\Delta_x, p_2\Delta_y)} \\ \vdots & & \vdots \\ b^{(1)}{}_{((p_1+mL)\Delta_x, p_2\Delta_y)} & \cdots & b^{(N)}{}_{((p_1+mL)\Delta_x, p_2\Delta_y)} \\ \vdots & & \vdots \end{bmatrix} \begin{bmatrix} f^{(1)}{}_{(p_1\Delta_x, p_2\Delta_y)} \\ \vdots \\ f^{(N)}{}_{(p_1\Delta_x, p_2\Delta_y)} \end{bmatrix} = \begin{bmatrix} \mu_{(p_1\Delta_x, p_2\Delta_y)} \\ \vdots \\ \mu_{((p_1+mL)\Delta_x, p_2\Delta_y)} \\ \vdots \end{bmatrix} \qquad [2]$$

In this 2D example, $b^{(n)}$ and μ represent, respectively, B1 field distribution and the target excitation profile. $f^{(n)}$ represents the periodic pattern associated with the nth RF pulse, with the period being $L\Delta_x$ as set by the coarse k_x-direction sampling. Design of SAR-reduced parallel excitation pulses is then a problem of minimizing ξ subject to a linear constraint that is given by a collection of Eq. [2]-type equation sets. With a pixel size chosen to match the spatial resolution requirement of the target profile, $f^{(n)}$ is related to the nth RF pulse's samples by Fourier transform. Using Parseval's theorem, ξ can thus be written as a quadric form in samples of $f^{(n)}$ and the optimization problem can be equivalently stated as a set of independent smaller optimization problems: minimize $\mathbf{f}_{p_1,p_2}^* \mathbf{\Phi} \, \mathbf{f}_{p_1,p_2}$ subject to Eq. [2]: $\mathbf{C}_{p_1,p_2} \mathbf{f}_{p_1,p_2} = \boldsymbol{\mu}_{p_1,p_2}$. Each of these sub-problems is solved by

$$\mathbf{f}_{p_1,p_2} = \mathbf{\Phi}^{-1} \mathbf{C}_{p_1,p_2}^* \left(\mathbf{C}_{p_1,p_2} \mathbf{\Phi}^{-1} \mathbf{C}_{p_1,p_2}^* \right)^{-1} \boldsymbol{\mu}_{p_1,p_2}, \quad \text{resulting in} \quad \xi_{\min} = 1/P \sum \boldsymbol{\mu}_{p_1,p_2}^* \left(\mathbf{C}_{p_1,p_2} \mathbf{\Phi}^{-1} \mathbf{C}_{p_1,p_2}^* \right)^{-1} \boldsymbol{\mu}_{p_1,p_2}. \qquad [3]$$

It can be shown there is a high degree of symmetry between the RF pulse and the SENSE recon solutions. In particular, analogous to SENSE SNR, the impact of transmit-SENSE acceleration on SAR may also be examined one set of "coupled" pixels (i.e., the pixels involved in the assembly of a $\mathbf{C}_{p1,p2}$ matrix) at a time and described with a ratio of the set's contribution to SAR between an accelerated case and its unaccelerated counterpart:

$$\delta SAR^{accelerated} \big/ \delta SAR^{unaccelerated} = g_t^2 R \text{ , where } g_t = \left[\boldsymbol{\mu}_{p_1,p_2}^* \left(\mathbf{C}_{p_1,p_2} \mathbf{\Phi}^{-1} \mathbf{C}_{p_1,p_2}^* \right)^{-1} \boldsymbol{\mu}_{p_1,p_2} \Big/ \boldsymbol{\mu}_{p_1,p_2}^* \left(\mathrm{DIAG}\left(\mathbf{C}_{p_1,p_2} \mathbf{\Phi}^{-1} \mathbf{C}_{p_1,p_2}^* \right) \right)^{-1} \boldsymbol{\mu}_{p_1,p_2} \right]^{1/2} \qquad [4]$$

In Eq. [4], DIAG is an operator that sets to zeros all of a matrix's off-diagonal entries; the appearance of R, the acceleration factor, reflects the usual scaling associated with shortening transmit duration while maintaining flip angle; and g_t captures the additional impact of acceleration. While the g_t-factor has a keen dependency on the target profile, a conclusion that generally holds is that the largest eigenvalue of the inverse matrix in the nominator is always ≥ that of the inverse matrix in the denominator, implying certain SAR penalty in the worst case and possible penalty in others.

Parallel excitation of a ∅24cm uniform cylinder inside an 8-element transmit array (Fig.1a) were simulated. The elements were distributed azimuthally on a ∅28cm shell. 2D pulses for achieving a flat target profile with various acceleration factors were calculated based on Eq. [3], and corresponding g_t maps (one value for each set of "coupled" pixels) were further computed (Fig. 1b-c). The simulation was repeated for a second transmit array that was of the same overall geometry but with wider and overlapped elements. In terms of ξ_{min}/R, the first array outperformed the second in the accelerated cases but slightly under-performed at R=1. Both arrays did better at R=4 than at R=1. Unlike SENSE reconstruction where noise correlation between "coupled" pixels does not impact perceived SNR, the total dissipated RF energy in parallel excitation is generally affected by the (B1 profile- and gradient trajectory-controlled) "pixel coupling", which, depending on how well it goes with a given target profile, may influence SAR in either direction. The present method could be potentially useful for optimizing excitation pulses and/or transmit arrays in practice, and also of significance, might shed lights on what the ultimate intrinsic SAR is and ways to approach it with parallel transmit.

1. K.P. Pruessmann, et al., *MRM* 42:952-962, 1999. 2. Y. Zhu, *MRM* 51:775-784, 2004. 3. Y. Zhu, *12th ISMRM*, p 331, 2004.