Estimation of the cross-terms for all b-matrix elements with an isotropic phantom

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Introduction

Quantitative diffusion tensor measurements have become a powerful tool to investigate the tissue microstructure. In order to compare DTI results measured with different protocols and on different scanners, an accurate calibration and exact DTI evaluation are needed. For correct DTI calculations the b-matrix has to be determined exactly, taking into account all diffusion weighting (DW) and imaging gradients. However, the required exact details of a pulse sequence are often unavailable for a routine user and the cross terms are ignored. Elements of the b-matrix are calculated according to the diffusion encoding (DE) direction and the given "nominal" b-factor. In this work we present a new method for estimation of the cross terms between imaging and diffusion encoding gradients and therefore improve the accuracy of DTI calculations.

Methods/Theory

Diagonal elements of b-matrix can be calculated using:

$$b_{ii}^{full} = \gamma^2 \int_0^{TE} \{[\int_0^t G_i(\tau)d\tau]^2 + [\int_0^t g_i(\tau)d\tau]^2 + 2[\int_0^t G_i(\tau)d\tau] * [\int_0^t g_i(\tau)d\tau]\}dt \qquad \text{(Eq.1)},$$

where $G_i(t)$ describes the shape of DW gradient and $g_j(t)$ describes the shape of imaging gradient along the axis i. The first term is a "nominal" b-matrix element b_{ii}^{DW} due to DW gradient only. The 2nd term corresponds to contributions from imaging gradients and the 3rd represents the cross term (CT) between imaging and DW gradients. The 2nd term is usually small and can be neglected since it is the same for all DW and non-DW images. The CT can be significantly large. As a rule the shapes $G_i(t)$ of DW gradients along different axes $i=x,y,z$, are equal ($=G_0(t)$) and just scaled with factors DE_i^k corresponding to DE step k. If DW gradients are zero during imaging gradients and vice versa (which is normally the case) the equation for effective b-factor can be simplified to:

$$b^{eff} \approx \sum_{i=x,y,z}(b_{ii}^{DW} + DE_i^k \int_0^{TE} 2\gamma^2 \{[\int_0^t G_0(\tau)d\tau] * [\int_0^t g_i(\tau)d\tau]\}dt) = \sum_{i=x,y,z} b_{ii}^{DW} + \sum_{i=x,y,z} DE_i{}^k b_{ii}^{CT1} \qquad \text{(Eg.2)},$$

where b_{ii}^{CT1} is the cross term with the DW gradient $G_0(t)$ scaled by factor $DE_i^0 = 1$. The cross terms b_{ii}^{CT1} in Eq.2 depend linearly on the amplitude of the DW gradient and therefore they it can be found from signal intensities I_k and I_0 by linear fit to the following equation:

$$\ln(I_k / I_0)/D = -(\sum_{i=x,y,z} b_{ii}^{DW} + DE_x^k b_{xx}^{CT1} + DE_y^k b_{yy}^{CT1} + DE_z^k b_{zz}^{CT1}) \qquad \text{(Eg.3)},$$

I_k should be measured in an isotropic phantom, and I_o is measured at $b^{eff} = 0$. Similar calculations for off-diagonal elements give the following result: $b_{ij}^{Full} = b_{ij}^{DW} + 1/2(DE_j{}^k b_{ii}^{CT1} + DE_i{}^k b_{jj}^{CT1})$ (Eq.4).

Hence the b-matrix can be completely estimated from DW measurements on an isotropic phantom.

Methods/Experimental

MR measurements were accomplished on a 3T Siemens Trio MR scanner with a standard product double echo DWI sequence [1], which was extended to arbitrary number and directions of DE gradients. For the calibration measurements a water filled phantom was used. The number of DE directions was varied from 12 to 61, the b-factor was up to 1000 mm^2/s, slice thickness 2.5 mm, 2x2 mm^2 in plane resolution and TR/TE 5000/105 ms, 4 averages. The temperature of the phantom was controlled by a thermometer. The temperature corrected ADC constant of water [2] was used for calculations. The exact shape of the gradients was generated with the Siemens sequence simulation tool and the exact b-matrix was calculated by numeric integration of Eq.1.

Results

Fig.1 presents the simulation of the effective b-factor. Fig.2 shows the fit according Eq.3 of the DW data, measured in the water calibration phantom (note: the imaging protocol was different from simulation in Fig.1). Table 1 presents the "nominal", exact calculated and experimentally estimated b-matrix.

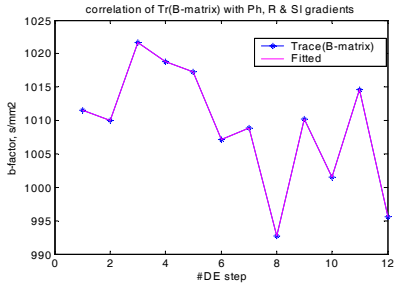

Fig.1 Simulation of the effective b-factor in double echo DW using 12 DE directions. Fit was done according Eq.(2), for details see Theory section.

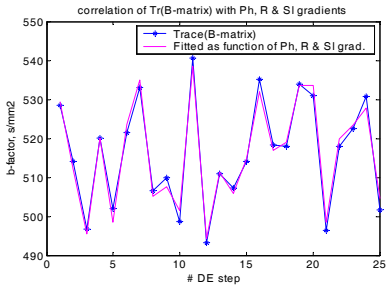

Fig.2. b-factor estimated from DW signal in water phantom and the fit according Eq. (3).

A)	207.09	0	-414.17
	0	0	0
	-414.17	0	828.34
B)	194.88	-2.85	-401.62
	-2.85	0	5.70
	-401.62	5.70	826.96
C)	197.32	-2.28	-404.13
	-2.28	0	4.56
	-404.13	4.56	827.24

Table 1. b-matrix for DE = [-0.45(R), 0(Ph) 0.89(Sl)], A – calculated according to DE direction, B - calculated exactly according to shape of all gradients, C – estimated from the measurement on a water phantom

Discussion

The proposed method can be useful for many clinical sites, which have no access to the pulse sequence details and therefore cannot calculate the b-matrix exactly. In such case the b-matrix estimations have to be done for each clinical protocol. The method is equally well applicable for double echo DW and for 'classical' spin echo DW.

Small mismatch between fitted and measured data in Fig.2 can be explained by imperfections of the gradients and partial overlapping of DW and imaging gradients during ramps. The method can be applied also for the 6 DE directions, but we get more accurate results for 12 and more DE directions. Noise in DWI data should be smaller than signal variation between different DE steps. For b-factor ≤1000 mm^2/s such SNR can easily be achieved by averaging a few times. For extreme high b-values, substances with lower diffusion constant are more preferable. Mismatch in gradient calibration factor along some axis would disturb linearity of Eq. 3 and this effect can be used for accurate gradients calibration.

References

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2. Holz M et al. Phys Chem Chem Phys 2000;2:4740-4742.