Spatially Variant GRAPPA

E. G. Kholmovski[1], D. L. Parker[1]

[1]Utah Center for Advanced Imaging Research, Department of Radiology, University of Utah, Salt Lake City, Utah, United States

Introduction: GRAPPA [1], a widely used technique for parallel MRI (P-MRI), has serious limitations for image reconstruction from highly undersampled data. Recently, two algorithms, GARSE [2] and MCMLI [3], have been proposed to resolve this GRAPPA limitation by exploiting all available data dimensions to find reconstruction coefficients that allow more reliable reconstruction for high reduction factors. However, inclusion of additional data dimensions in reconstruction substantially increases the number of reconstruction coefficients resulting in low computational efficiency of these new algorithms. Furthermore, image reconstruction speed for the new algorithms is also reduced because the Fourier transform in the readout direction cannot be applied immediately after each readout acquisition. In this abstract, a novel algorithm is presented that achieves equivalent image quality from highly undersampled data as the multi-dimensional auto-calibrating techniques but has computational efficiency comparable to that of GRAPPA.

Theory and Methods: Image $s_i(r)$ acquired by the i–th coil element can be described by $s_i(r)=c_i(r)f(r)$, where $f(r)$ is the imaged object, $c_i(r)$ is the i-th coil sensitivity (i=1:Nc, Nc – the number of coils). In k-space, the equation is given by $S_i(k)=C_i(k) \otimes F(k)$, where the symbol $\otimes$ denotes convolution. For simplicity of presentation, the proposed reconstruction method will be described for the 2D imaging case ($s_i(r)=s_i(x,y)$) with data undersampling along the phase encoding dimension k_y. Generalization to 3D imaging and undersampling in both phase-encoding dimensions is straightforward.

In 2D P-MRI with data acquisition on a regular Cartesian grid, only a fraction of k_y views required by the Nyquest criterion is acquired. For each acquired k_y view, all required k_x locations are sampled. Fourier transform along k_x can be used to reconstruct an intermediate image estimate: $S_{x,i}(k_y)=C_{x,i}(k_y) \otimes F_x(k_y)$, where index x=1:Nx, Nx is the number of pixels spanning the image field-of-view (FOV) in the x (readout) direction. Thus, the 2D P-MRI reconstruction problem can be subdivided into Nx 1D reconstruction problems. The last equation can be expressed in matrix form $\mathbf{S}_{x,i}=\mathbf{C}_{x,i}\mathbf{F}_x$ for each individual coil dataset. Combining all coil equations, the matrix equation describing P-MRI with k-space sampling on a Cartesian grid is given by $\mathbf{S}_x=\mathbf{C}_x\mathbf{F}_x$ with x=1:Nx.

In the case of k-space undersampling, the vector $\mathbf{S}_x$ and matrix $\mathbf{C}_x$ can be presented as $\mathbf{S}_x=\left[\mathbf{S}'_x \ \hat{\mathbf{S}}_x\right]^{\mathrm{T}}$ and $\mathbf{C}_x=\left[\mathbf{C}'_x \ \hat{\mathbf{C}}_x\right]^{\mathrm{T}}$, where $\mathbf{S}'_x$ and $\hat{\mathbf{S}}_x$ denote vectors of known and unknown data, respectively, and $\mathbf{C}'_x$ and $\hat{\mathbf{C}}_x$ are the corresponding sub-blocks of the matrix $\mathbf{C}_x$. A system of two matrix equations with unknown $\mathbf{F}_x$ and $\hat{\mathbf{S}}_x$ can be constructed as $\mathbf{S}'_x=\mathbf{C}'_x\mathbf{F}_x$ and $\hat{\mathbf{S}}_x=\hat{\mathbf{C}}_x\mathbf{F}_x$. The solutions to the system can be found when the pseudo-inverse of $\mathbf{C}'_x$ exists and are given by $\mathbf{F}_x=(\mathbf{C}'^H_x\boldsymbol{\Psi}^{-1}\mathbf{C}'_x)^{-1}\mathbf{C}'^H_x\boldsymbol{\Psi}^{-1}\mathbf{S}'_x$ and $\hat{\mathbf{S}}_x=\mathbf{A}_x\mathbf{S}'_x$, where $\mathbf{A}_x=\hat{\mathbf{C}}_x(\mathbf{C}'^H_x\boldsymbol{\Psi}^{-1}\mathbf{C}'_x)^{-1}\mathbf{C}'^H_x\boldsymbol{\Psi}^{-1}$ and $\boldsymbol{\Psi}$ is a matrix describing the coupling and noise correlation between the receiver coils. The first equation describes image reconstruction from the undersampled P-MRI data when coil sensitivities are known [4,5]. The second equation shows that missing data in the individual coil datasets can be found by linearly combining the acquired data:

$$S_i(x,\hat{k}_y)=\sum_{j=1:Nc}\sum_{k'_y} a(i,j,x,k'_y)S_j(x,k'_y) \qquad [1]$$

where k'_y ($\hat{k}_y$) denote the acquired (missing) k_y views. The coil sensitivities $c_i(r)$, which vary slowly in image space, can be described by a small number of Fourier terms. Thus, only $S_j(x,k'_y)$ (j=1:Nc) in k_y locations adjacent to $\hat{k}_y$ should be used to estimate $S_i(x,\hat{k}_y)$:

$$S_i(x,\hat{k}_y)=\sum_{j=1:Nc}\sum_{k'_y\in\Omega(\hat{k}_y)} a(i,j,x,k'_y)S_j(x,k'_y) \qquad [2]$$

where $\Omega(\hat{k}_y)$ is a neighborhood of $\hat{k}_y$. The GRAPPA algorithm [1] can be presented in the analogous form:

$$S_i(x,\hat{k}_y)=\sum_{j=1:Nc}\sum_{k'_y\in\Omega(\hat{k}_y)} b(i,j,k'_y)S_j(x,k'_y) \qquad [3]$$

The main distinction between this new algorithm, Spatially Variant (SV)-GRAPPA, and GRAPPA is how the reconstruction coefficients depend on the x coordinate. In SV-GRAPPA, the reconstruction coefficients $a(i,j,x,k'_y)$ are spatially varying and change according to local coil sensitivity characteristics. Whereas GRAPPA reconstruction coefficients $b(i,j,k'_y)$ are spatially invariant and based on global coil sensitivities characteristics. Thus, GRAPPA is only optimal when coil sensitivities are independent of the x coordinate. This assumption is not true for typical receiver coils used in MRI.

In case of P-MRI with sampling on the regular 2D Cartesian grid, coefficients $a(i,j,x,k'_y)$ can be evaluated from auto-calibrating k-space lines using the fitting procedure proposed for GRAPPA [1]. To get relevant reconstruction coefficients, the fitting should be highly over-determined because auto-calibrating data are contaminated by noise. In SV-GRAPPA, the number of auto-calibrating data for each x position is small. Therefore, the fitting can be ill determined for SV-GRAPPA. To resolve this problem, two approaches were considered. Both of them are based on the fact that coil sensitivities can be described by slowly varying functions in the image domain, and, therefore, $a(i,j,x,k'_y)$ should be also slowly varying relatively to the x coordinate. In the first approach, auto-calibrating data are divided in overlapping blocks along the x direction, a set of reconstruction coefficients is calculated for each block, and the sets are interpolated to find $a(i,j,x,k'_y)$ for all x values. In the second approach, $a(i,j,x,k'_y)$ are presented by a few terms of the Fourier series:

$$a(i,j,x,k'_y)=\sum_m c(i,j,m,k'_y)\exp(i\gamma xm) \qquad [4]$$

where $\gamma=2\pi/Nx$, m=-(Nm-1)/2, -(Nm-3)/2, ..., (Nm-1)/2, and Nm is an odd integer. At first, the $c(i,j,m,k'_y)$ are estimated using the fitting procedure with all available auto-calibrating data (highly over-determined problem). Then, the coefficients $a(i,j,x,k'_y)$ are calculated according to Eq. [4].

SV-GRAPPA needs the same number of computational operations as conventional GRAPPA to reconstruct the missing k-space lines when the number of the reconstruction coefficients is the same for both techniques and the coefficients are known. The number of computations required for SV-GRAPPA coefficients estimation strongly depends on the number of x blocks and their width for the first approach and the factor Nm for the second approach. It is typically higher than the number of computations needed for GRAPPA coefficients estimation.

Results: To test the proposed technique, MRI studies were performed on a Siemens Trio 3 Tesla system using the eight-channel head coil. Typical dependence of SV-GRAPPA reconstruction coefficient on x coordinate is shown in Fig. 1. For comparison, the corresponding GRAPPA coefficient value is plotted. It is obvious that SV-GRAPPA coefficient more accurately reflects a complexity of coil sensitivity maps than GRAPPA coefficient. This allows SV-GRAPPA to achieve significantly better image quality and lower RMS errors than GRAPPA, especially for high reduction factors (Fig. 2).

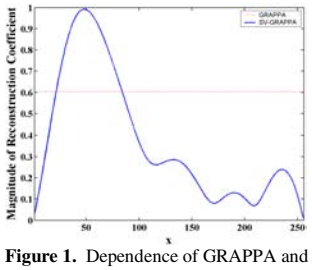
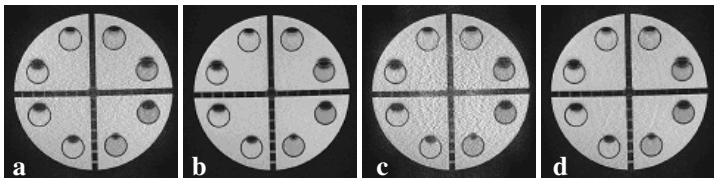


Figure 1. Dependence of GRAPPA and SV-GRAPPA reconstruction coefficient magnitude on x coordinate.

Conclusion: A novel auto-calibrating technique for P-MRI has been developed. SV-GRAPPA, like multi-dimensional auto-calibrating techniques, is based on the fact that sensitivities of receiver coils vary in all spatial dimensions. Therefore, the reconstruction coefficients should also vary spatially and be adjusted according to local coil sensitivity characteristics to get optimal image reconstruction from highly reduced P-MRI data.



106.8 **53.7** **186.6** **85.6**

Figure. 2. Phantom images reconstructed by GRAPPA (a,c) and SV-GRAPPA (b,d). Undersampling rate, R, for (a,b) is equal 3, for (c,d) R=4. Numerical values given below the images indicate the corresponding RMS errors. SV-GRAPPA reconstruction coefficients were estimated using the first approach with 12 blocks.

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