A Fast Entropy Minimization Algorithm For Bias Field Correction in MR Images

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Introduction: Intensity inhomogeneity in MR images, primarily generated by sensitivity of the radio frequency coil, is one of the major problems in the digital analysis of MR data. The inhomogeneity correction with no independent bias field or bias free image measurements is a blind separation problem. To separate the bias field with the true image data, many criterions have been proposed. Minimizing Shannon entropy is a nature criterion for many imaging restoration problems and can be used for the blind separation of the bias field and true image data without any assumptions on the image itself. However the entropy minimization is a non-linear problem, and the entropy is usually not a explicit function of the spatial parameters, therefore the minimization is usually performed with function value based algorithm[1][2]. In this work, we propose a new approach for the entropy minimization. This approach is based on using a non-parametric Parzen window [3] to estimate the probability distribution function (PDF) of the image. A differatiatable Gaussian Kernel was chosen for the Parzen window estimation. By this approach, the derivatives of the entropy to the spatial parameters of the bias field can be directly calculated and the minimization can be performed in a more efficient conjugate gradient algorithm [4].

Method: The bias field corrupted image can be corrected by multiply a correction function *B(x,y,z)* to the measured image *v(x,y,z)* [1].

$$u(x,y,z) = B(x,y,z)v(x,y,z)$$

where the noise has been ignored. The B(x,y,z) can be modeled as a polynomials:

$$B(x,y,z) = \sum_{i,j,k} a_{i,j,k} P_i(x) P_j(y) P_k(z) = \sum_{i,j,k} a_{i,j,k} f_{i,j,k}$$

in above formula, the $P_n(x)$ is the nth order Legendre polynomial and the $a_{i,j,k}$ is the coefficients. The optimization on the coefficients can be done by minimizing the entropy of the image *u(x,y,z)* under the constraint of mean preserving[1]. The entropy can be written:

$$H = -\sum_u p(u) \log p(u)$$

in (3) the p(u) is the probability distribution function (PDF) of image u. Using non-parametric Parzen window approach the calculation of the pdf:

$$p(u) = \frac{1}{N} \sum_{x,y,z} g_\sigma(u - u(x,y,z)) = \frac{1}{N} \sum_{x,y,z} g_\sigma(u - B(x,y,z)v(x,y,z))$$

where g(x) is an kernel function with width d, and N is the total number of measured voxels. The kernel function can be chosen as Gaussian function because of its highly differentiability.

$$g_\sigma(x) = \frac{1}{\sqrt{\pi}\sigma} e^{-\frac{x^2}{\sigma^2}}$$

The derivatives of the entropy to the coefficients of the polynomial can be calculated:

$$\frac{\partial p(u)}{\partial a_{i,j,k}} = \frac{1}{N} \sum_{x,y,z} g'_\sigma(u - B(x,y,z)v(x.y.z))(f_{i,j,k} - \bar{f}_{i,j,k})v(x,y,z)$$

$$\frac{\partial H}{\partial a_{i,j,k}} = -\sum_u (\log p(u) + 1) \frac{\partial p(u)}{\partial a_{i,j,k}}$$

With the information of gradient, the minimization of the entropy can be performed by an iterative algorithm called conjugate gradient method [3].

	Jcv(W/G)	*cv*(W)	*cv*(G)
Uncorrected	144.0	7.5	8.5
Corrected(Powell)	94.5	5.0	5.4
Corrected(This)	93.5	4.7	5.1

Table 1 Correction Results

Results: The test of the algorithm is performed on a clinical obtained PD weighted 3D image data set (SIEMENS 1.5T symphony) containing 22 slices, each slice has FOV 210 mm with 256x256 resolutions, and the slice thickness is 6 mm. The original image was corrupted by a typical phased array coil profile. The algorithm was implemented in C and run on a SGI fuse work station. For comparison, the entropy minimization algorithm based on [1], which uses Powell's method to minimize the entropy, was also implemented. In the algorithm, a 5th order 3D Legendre polynomial was used as the correction function *B(x,y,z)*. The total fitting coefficients for the bias correction was 54. The efficiency of the algorithm was evaluated by counting the number of times the algorithm update entropy. For the accuracy study, a segmented white matter and gray matter tissue map was obtained by an automatic segmentation self-organizing map algorithm [5]. The accuracy of the algorithm was evaluated by calculation of the joint covariance between the intensity levels of white and gray matters (*jcv*) as well as the covariance within white matter and gray matter (*cv*). Fig. 1 (a) shows a single slice uncorrected image and (b) and (c) are the corresponding corrected images with this algorithm and Powell's algorithm respectively. Table 1 lists the number of time for the algorithm to update entropy. Table 2 lists the *jvc* and *cv* for the uncorrected image and the corrected image with different algorithm. We can visualize the correction effect from Fig.1. The new method performs better than the old implementation. The number of times the algorithm updating the entropy is 1054 for the new method and 39201 for the Powell's method. This test shows that the new algorithm is much more efficient than the old one. From Table 1, we find that the overall image qualities of the corrected image using gradient based algorithm are better or equivalent to the Powell's method.

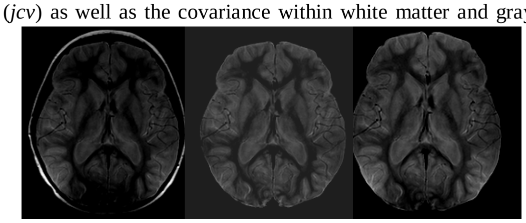

Fig. 1 Single slice clinical PD weighted image. From left to right: uncorrected image, corrected by this method and corrected by Powell's method

Discussions: The new method uses entropy gradient information to guide the search direction in multidimensional space. The test results show that it is more efficient than function value based Powell's method.

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