A Matrix Framework for General Non-Affine Motion Correction in Multishot Acquisitions

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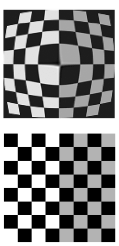
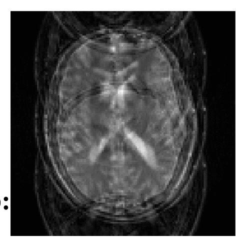
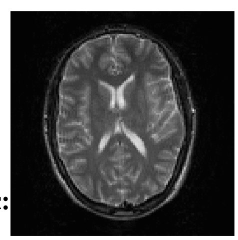
Theory: We are interested in reconstructing MR images that have been corrupted by motion which took place between the shots of a multi-shot acquisition. Images acquired in the Fourier domain, and which have been motion corrupted are in general corrected by using two facts about Fourier transforms: the Fourier transform of a translated image is the original data with a phase shift, and the Fourier transform of a linearly transformed image is a linear transform of the Fourier transform of the original image. Two problems appear when trying to generalise to non-affine motions, or motion acquired in multiple shots. First, the Fourier data acquired at different times, i.e., k-space positions, is not consistent, which causes the ghosts. Secondly, no general statement holds for the Fourier transform of an image corrupted by nonrigid motion, in terms of transforms of the coordinates in the underlying space: we can't simply find a nonrigid motion of k-space corresponding to a nonrigid motion of image space. For this we develop a formalism which, given a description of the motion, allows reconstruction of an uncorrupted image (if adequately sampled). This method works for *any* type of motion, including nonlinear deformations and there are no specific requirements on the motion, such as a limitation to the phase encode (PE) direction. Noting that discrete samples of k-space are obtained at distinct times *t=0…n_shots-1*, (sampling matrix $\boldsymbol{A}_t$) and define $\boldsymbol{a}_t = \boldsymbol{F}^H \boldsymbol{A}_t \boldsymbol{F}$ ($\boldsymbol{F}$ Fourier matrix), which is the image domain aliasing matrix. The essential point of our method is that any spatial transformation induces a *linear* transformation on the space of images, which we can write as a matrix $\boldsymbol{u}_t$. Expressing the uncorrupted image $\boldsymbol{s}_0$ and the ghosted image $\boldsymbol{s}$ as arrays we can write a general expression $\boldsymbol{s} = \left(\sum_{t=0}^{n_s-1} \boldsymbol{a}_t \boldsymbol{u}_t\right)\boldsymbol{s}_0 =: \gamma \cdot \boldsymbol{s}_0$. This expresses the relationship between the motion corrupted and required motion free images as a linear transformation, suggesting $\boldsymbol{s}_0$ may be obtained from $\boldsymbol{s}$ by inversion of γ. Although the matrix γ is in general singular, we can use a conjugate gradient least squares solver for the linear system, which converges even for a singular system, and requires only matrix-vector multiplications. An advantage of γ is that multiplication by it can be implemented without ever building the matrix, but only using FFTs, image transformations, and subsampling. In this way, this method (in spirit comparable to [1] [4]) can be implemented without excessive computational costs.

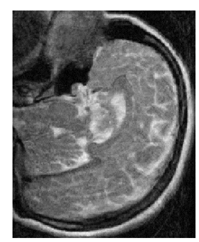
Results: To test the method different types of rigid and nonrigid motions were simulated (pulsation, piecewise translations or rotations) and corrections applied either using knowledge of the motion or by minimising an autofocus energy**.** Figure **1a**) shows checkerboards depicting a non-uniform radial expansion applied to shots 2 and 3 with return to baseline in shots 1 and 4 of a simulated 4 shot brain image. The motion is $\boldsymbol{r} \rightarrow \boldsymbol{r}^{1+\alpha_t}$, where $\boldsymbol{r}$ is the distance of a pixel to the center of the image, and α_t a parameter controlling the deformation (here $\alpha_{1,4} = 0, \alpha_{2,3} = 0.5$). The corrupted image is shown in Fig. **1-b)** (left) and the correction applied with knowledge of the motion is shown in Fig. **1-c)**. Figures **2-a)**,**b)** show real data correction of a nodding head. The motion corrupted image **2-a)** was created by mixing shots from the head in 8 different positions, from which the motion, (here affine was considered appropriate) was manually estimated. Note how the gyri have become visible in the corrected image **2-b).**

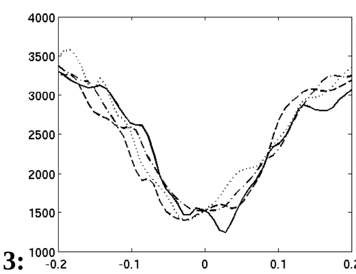
1-a: 

1-b: 

1-c: 

2-a: 

2-b: 



3:

Discussion: Unknown Motion. In practice the motion is not usually known, although various auto-focus energies have been used to estimate motion histories from the acquired data (see e.g. [2]). Previous methods, however, have generally been limited by the use of inexact or limited motion correction algorithms (for example applying the inverse motion to sets of k-space lines acquired at single time points) so that ghosts are not eliminated completely, even when the correct motion history is applied. The current method does not suffer this limitation. We have also used an autofocus energy which measures the consistency between different (fully sampled) coil 'views' [3]. The rationale is that when the image is not ghosted, the coil views are more consistent. We write $\boldsymbol{s}_i$ for the image measured by the coil with sensitivity $\boldsymbol{c}_i$, and define the energy as the sum over all pairs of | $\boldsymbol{s}_i / \boldsymbol{c}_i - \boldsymbol{s}_i / \boldsymbol{c}_i$ |. This energy is displayed in Fig. **3**), for simulated images deformed as in Fig **1-a)**, where each line corresponds to varying the exponent α_t away from the correct value, for each of the four shots.

Conclusion. We have established a formula for describing motion in MR images that is applicable to arbitrary motions (and in fact also to arbitrary sampling patterns). This formalism allows correction of motion artifacts using standard optimisation strategies and initial results are promising. This extends [3] by lifting the previous restriction to 1D components of motion. This framework is an advance on previous work in the complexity of the motion it can correct, and the computational efficiency of the algorithm.

References [1] Pruessman, et al, *Magn.Res.Med*, **46**:638-651, 2001 [2] Atkinson et al, *Magn Res Med* 47:777-786,1999 [3] Atkinson et al *Magn.Res.Med*. **52**:825-830, 2004 [4] Bydder, M. ISMRM 2004, 2124 1