Effects of Phase Perturbations in SENSE Coil Calibration

C. Lew[1], D. Spielman[1], F. Chan[1], R. Bammer[1]

[1]Stanford University, Stanford, CA, United States

Introduction: SENSE (1) requires the use of accurate coil sensitivities to produce an accurate reconstruction. The acquisition in image space can be represented by $\mathbf{S} = \mathbf{C}\rho$ [1] where **S** is an mx1 vector representing the acquired dataset in image space for a set of m coils, **C** is an mxn matrix representing the coil sensitivities, and ρ is an nx1 vector representing the unaliased magnetization for a set of n pixels. If the decoding coil sensitivity matrix is a perfect measurement, the reconstruction can be computed in a minimum least squares sense with the result that $\rho_{recon} = (\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{S}$. The g-factor for this reconstruction is $g_p = \sqrt{((\mathbf{C}^H\mathbf{C})^{-1})_{p,p}(\mathbf{C}^H\mathbf{C})_{p,p}}$, where p denotes the pth pixel. In this abstract, we investigate the case when the coil sensitivities have phase errors in their measurements and their effects on the magnitude and phase SENSE reconstructions.

Theory: We can attribute the phase of an image to many sources. A non-exhaustive list includes field inhomogeneity, chemical shift, receiver time delays, flow encoding, and coil phase. When the decoding coil sensitivities are calculated, two typical examples are to divide the individual coil calibration measurements by the body coil and by the sum-of-squares. If the calibration scans are performed at separate instances, i.e. different scan or cardiac phase, then both the SENSE acquisition and calibration acquisition will have phases unique to their own scan. If we were to divide by the complex body coil image, phases common to the body coil image and individual coil calibration image such as from off-resonance would cancel, and the decoding coil sensitivity would have this difference in phase represented in its matrix. If we were to divide by the sum-of-squares image, the decoding coil sensitivity would have the phase acquired from the individual coil calibration scan. We can represent these perturbations by $\tilde{C}_{rs} = C_{rs}|D_s|\exp(i\angle D_s)$. Note that D_s represent image dependent variations. The decoding coil matrix is now $\tilde{\mathbf{C}} = \mathbf{CD}$, where **D** is an nxn invertible diagonal matrix with n being the number of aliased pixels and $D_{ss}=D_s$. Let $\tilde{\rho}_{recon}$ be the modified reconstructed magnetization. By equation [2], $\tilde{\rho}_{recon} = \mathbf{D}^{-1}\rho_{recon}$. $\tilde{\rho}_{recon}$ is the SENSE reconstruction multiplied by the inverse of the image dependent variation. In other words, common phases to the coil sensitivity and to the SENSE acquisition will not manifest in the SENSE reconstruction, but those phases in the coil sensitivity that are not accrued during the SENSE acquisition appear as negative phases in the SENSE reconstruction. Furthermore, the perturbed g-factor would be the same as the original g-factor, or $\tilde{g}_p = g_p$ [3]. Ignoring random coil measurement noise, the perturbed SENSE equation performs the same as the unperturbed case when reconstructing magnitude images.

$$\begin{aligned}\tilde{\rho}_{recon} &= (\tilde{\mathbf{C}}^H\tilde{\mathbf{C}})^{-1}\tilde{\mathbf{C}}^H\mathbf{S}\\ &= ((\mathbf{CD})^H(\mathbf{CD}))^{-1}(\mathbf{CD})^H\mathbf{S}\\ &= \mathbf{D}^{-1}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\mathbf{S}\\ &= \mathbf{D}^{-1}\rho_{recon}\end{aligned} \quad [2]$$

$$\begin{aligned}\tilde{g}_p &= \sqrt{((\tilde{\mathbf{C}}^H\tilde{\mathbf{C}})^{-1})_{p,p}(\tilde{\mathbf{C}}^H\tilde{\mathbf{C}})_{p,p}}\\ &= \sqrt{(((\mathbf{CD})^H(\mathbf{CD}))^{-1})_{p,p}((\mathbf{CD})^H(\mathbf{CD}))_{p,p}}\\ &= \sqrt{((\mathbf{D}^H)^{-1}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{D}^{-1})_{p,p}(\mathbf{D}^H\mathbf{C}^H\mathbf{CD})_{p,p}}\\ &= \sqrt{(|D^{-1}_{pp}|^2(\mathbf{C}^H\mathbf{C})^{-1}_{p,p})(|D_{pp}|^2(\mathbf{C}^H\mathbf{C})_{p,p})}\\ &= \sqrt{(\mathbf{C}^H\mathbf{C})^{-1}_{p,p})(\mathbf{C}^H\mathbf{C})_{p,p}}\\ &= g_p\end{aligned} \quad [3]$$

Methods: A fully-sampled 256x256 FSE brain scan (TE/TR=3.2ms/100ms) with an 8-channel head coil was performed on a GE Signa 1.5T scanner. The scan was undersampled by 4 on computer, and noise was added. The fully-sampled image was used to create the coil sensitivities with division by a separate body coil scan. Next, Gaussian phase perturbation was added to the coil sensitivity. The g-factor and SENSE reconstruction was computed before and adding the phase perturbations to validate the mathematical analysis. Next, a phase-contrast scan with 20 cardiac phases was performed with a reduction factor of 2 on a 4-coil torso coil. A separate torso-coil and body-coil calibration were performed. These calibrations were done with phase contrast as well. 3 calibration cases were performed for each calibration type. Case 1 synchronizes each flow image of the calibration scan with that of the SENSE acquired scan. Case 2 synchronizes the first flow image of each cardiac phase as calibration for the corresponding pair of flow images for the SENSE scan. Case 3 uses the first flow image of the first cardiac phase as the calibration for the entire set of cardiac phases. A complex phase difference was used to create the flow measurement.

No phase perturbation

Phase perturbation

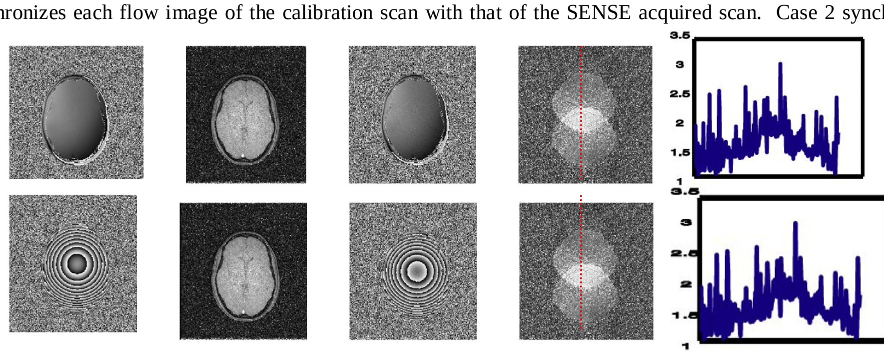

Fig 1 **a** **b** **c** **d** **e**

Results: Figure 1 shows the brain simulation results. The top row is the unperturbed acquisition, while the bottom row is the Gaussian perturbed acquisition. Column a) shows the calibration phase, b) shows the magnitude SENSE reconstruction, c) shows the phase SENSE reconstruction, d) shows the g-factor map, and e) shows the g-factor along the profile delineated by the red line. Figure 2 shows the flow measurements for the a) body coil and b) sum-of-squares.

Discussion: Figure 1 columns b), d), and e) clearly show that the g-factor and SENSE magnitude reconstruction are unchanged by phase perturbation. Sincecalibration with body coil phase is a division process, the phase of the SENSE reconstruction in c) is the same as the body phase in a) as expected. Furthermore, since the phase perturbation is added, the phase reconstruction is the inverse of the added phase. In figure 2, both graphs show that case 1 does not match case 2 and case 3. This can be inferred because, for case 1, each calibration cancels out the flow-encoded phase of the SENSE acquired scan. For the body coil, this is not as prevalent because the flow-encoded phase of the body scan has canceled out most of the torso coil calibration flow-encoded phase. In case 2 and 3, the same phase was perturbed at each flow image, and thus the complex phase difference will cancel out these phase perturbations. This result will be essential for future phase-contrast SENSE applications when self-calibration is considered. Note that in general, we may not know **D** and may have to consider it to be a random process. Since no assumptions about **D** were made except that it were invertible, future work would be to generalize this to magnitude errors. Also, coil-to-coil perturbation is another factor to be considered.

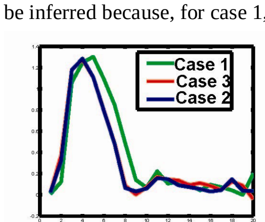

Fig 2a

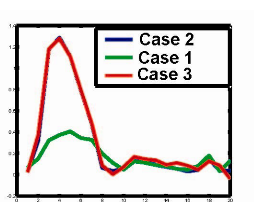

Fig 2b

Acknowledgments: Support was provided by GE Medical Systems, NIH grants RR09784, 1R01EB002771, and GM08412, Center of Advanced MR Technology at Stanford P41RR09784, the Oak Foundation, and the Lucas Foundation. **References:** 1. Pruessman, K.P., et al. MRM, 42:952-962, 1999.