Spiral SENSE Reconstructions Using Direct Matrix Inversion

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Introduction

K-space implementations of spiral SENSE reconstructions have been demonstrated elsewhere at this meeting (1). Conjugate gradient (CG) reconstructions (1) have been demonstrated to provide identical image reconstruction performance as the standard image space solution approach (2). In this study, we demonstrate that the CG algorithm can be effectively replaced by a direct matrix inversion to solve the k-space matrix equation, without affecting the quality of the reconstructions. The motivation for this strategy is that coil sensitivity maps have very small extensions in k-space, and the inversion can be decomposed into a series of extremely small matrices which can be easily solved with efficient algorithms. The feasibility of this strategy is shown here with experimental data from a quality assurance phantom data acquired at 1.5T.

Method and Materials

In k-space, the spiral SESNE reconstruction can be described by a matrix equation **BP**=**G**, with $\mathbf{B} \equiv \mathbf{C}^H\mathbf{DC}$ and $\mathbf{G} \equiv \mathbf{C}^H\mathbf{Dm}$, consisting of the k-space sensitivity matrix **C**, the k-space data **P**, and acquired spiral data **m**. A diagonal matrix **D** is used for compensating for the non-uniform sampling density of the spiral trajectories. The direct inversion to this equation is **P**=**AG**, with **AB**=**I**. Accounting for small extensions of the k-space sensitivity maps, the nonzero elements in a row of the matrix **A** or **B** concentrate in a small region (Fig.1), and, thus, the matrix **A** can be determined row-by-row, i.e. $\Sigma_{k'} B^*_{k'j} A_{kj} = \delta_{kk'}$, for the *k*th row, for example. These sub-matrix equations can be further simplified and downsized four times due to the complex conjugate symmetry of the matrices **A** and **B** (i.e. $\mathbf{A}^H=\mathbf{A}$ and $\mathbf{B}^H=\mathbf{B}$), and, thus, be efficiently and reliably solved with a regularized complex Cholesky decomposition (3).

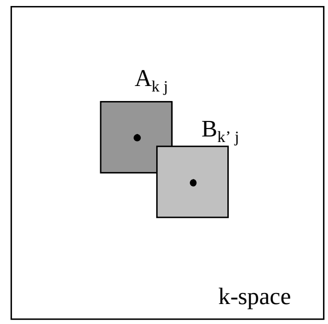

Fig.1:. *Schematic depiction of nonzero element distributions in the k^{th} row of the matrix **A** and in the k'^{th} row of matrix **B**.*

Phantom experiments were performed on a 1.5T whole body MRI scanner (GE Signa CV/i, Milwaukee, WI) using uniformly interleaved spirals and a 4-element coil array. Data collection parameters were 24 interleaves, 256×256 image matrix size, and 2872 pts/leaf. Fully sampled k-space data sets were also collected. The inversion was implemented on the 2× undersampled data set consisting of even spiral leaves. The k-space sensitivity map had a dimension of 12×12, and the size of the nonzero elements in a row of the matrix **A** was chosen as 10×10. The inversion method was programmed on UNIX C/C++.

Results and Discussion

Fig.2 illustrates the performance of the direct inversion method on the phantom data. Strong undersampling artifacts are evident in Fig.2a due to the size of the phantom (which completely filled the FOV). These artifacts have been successfully eliminated in Fig. 2b after the application of the proposed technique. Comparison of the proposed method with the image-space CG technique demonstrates a negligible 2.5% RMS error which once again confirms the effectiveness of the proposed approach (Fig 2c).

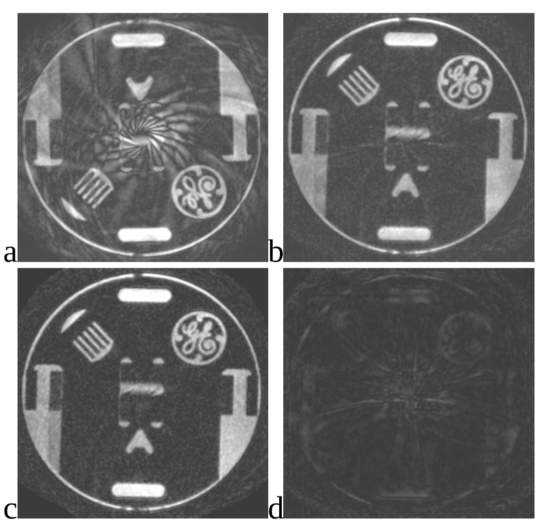

Fig.2:. *Phantom images: (a) before and (b) after inversion, (c) reference image, and (d) difference between (b) & (c). All images are displayed in the same window/level.*

The proposed direct inversion method has two major advantages over the CG iterative algorithm for k-space spiral SESNE. The matrix **A** only contains information about the sensitivity maps and the spiral trajectories, and thus can be pre-calculated. Because the sensitivity maps are slowly varying in space, this provides an opportunity to speed up reconstruction time by using the same sensitivity map, i.e., the matrix **A**, for neighboring slices of a 3D reconstruction. Furthermore, the computation complexity of the matrix **A** does not increase with the number of coils used in parallel imaging, making the method better suited for large scale parallel imaging than the conventional CG approach. The main drawback of the inversion method is that the reconstruction time could be long if one chooses to solve the sub-matrix equation for all grid points. Nevertheless, this run-time burden can be removed by performing Cholesky decompositions on representative grid points instead of the entire grid as it is done in approaches such as GRAPPA (4).

References:

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