Optimum k-Space Sampling for Parallel MRI with Large Acceleration

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Introduction

Parallel or sensitivity-encoded imaging methods, like SMASH [1], SENSE [2], use multiple receiver coils (with distinct spatial sensitivities) to simultaneously acquire phase-encoded data for an object at a sub-Nyquist sampling rate determined by the field-of-view (FOV) of the object. The density and number of k-space samples critically affects the resolution and artifacts in the reconstructed image, and the imaging time. In this paper we show how one can optimize the k-space sample locations to minimize the rms reconstruction error. The related theory was described recently [3], but this paper provides the first such experimental demonstration.

Theory

Imaging Equation : In parallel imaging experiment with N_c coils, the signal detected by the nth coil, with coil sensitivity $c_n(x,y)$ is :

$$s_n[k_x(m),k_y(p)] = \int f(x,y)c_n(x,y)e^{-j2\pi(k_x(m)x+k_y(p)y)}dxdy \quad + \quad \mathbf{v}_{nmp} \tag{1.1}$$

where *f(x,y)* is the desired object function; $\{k_x(m),k_y(p)\}$ is the k-space sampling location; and v_{nmp} is the additive zero-mean Gaussian noise. The discrete approximation to Equation (1.1) can be written as $\mathbf{s}_\Lambda = \mathbf{A}_\Lambda \mathbf{f} + v$ where vector **f** represents the object on a discrete grid; **s** is the observation vector and $\mathbf{A}_\Lambda$ is the matrix relating the two. Note that the observations **s** depend not only upon the object **f**, but also on the coil sensitivity profiles c_n 's and the set of k-space sample locations Λ .

Object Model : As in [3], the discretized object $f(m,n) = g(m,n)\,W(m,n)$ is modeled as a windowed version of a two-dimensional, zero-mean, stationary Gaussian random process *g(m,n).* The process *g(m,n)* is characterized by its power-spectral density, which captures how the energy in the image is distributed at different spatial frequencies. The spatial window *W(m,n)* reflects the finite region of support of the object. Both these quantities can be estimated from data that is collected during the pilot scan for estimating the coil-sensitivities, or alternatively by reference to an image database.

Error Metric : Using the above imaging and object model, it can be shown that optimal minimum mean square error (MMSE) reconstruction and the corresponding MSE are given by the following equations where $R_{\mathbf{ab}} = E[\mathbf{ab}^*]$ is the covariance matrix for vectors **a** and **b**:

$$\mathbf{f}^* = \underset{\hat{\mathbf{f}}}{\arg\min}\, E[\|\hat{\mathbf{f}} - \mathbf{f}\|^2] = R_{\mathbf{fs}_\Lambda} R^{-1}_{\mathbf{s}_\Lambda \mathbf{s}_\Lambda} \mathbf{s}_\Lambda \quad ; \quad MSE(\Lambda) = tr[R_{\mathbf{ff}} - R_{\mathbf{fs}_\Lambda} R^{-1}_{\mathbf{s}_\Lambda \mathbf{s}_\Lambda} R_{\mathbf{s}_\Lambda \mathbf{f}}] \tag{1.2}$$

The covariance matrices in Equation (1.2) can be computed from the object model and the set of k-space sample locations Λ [3].

Applications : The MSE depends upon the coil sensitivities c_n 'sand k-space sample locations Λ , but is independent of the actual observations and hence can be computed prior to data acquisition. Thus for a given set of RF coils, one can optimize the set of k-space sample locations to minimize the resultant MSE. The error metric can be modified, based on the application, to emphasize fidelity to different aspects (for example, edges) in the reconstructed image [3].

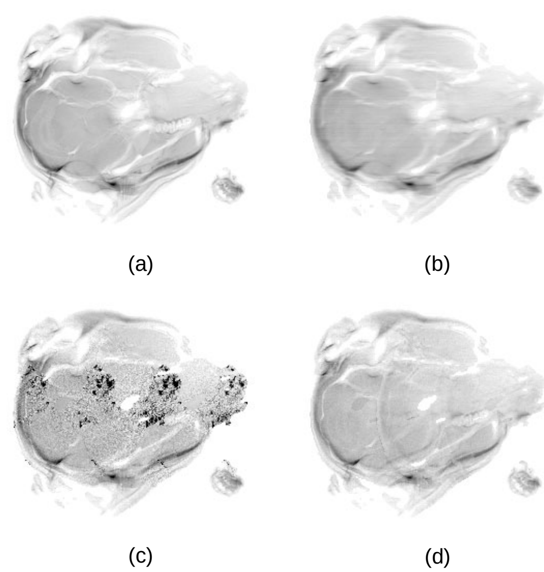
Method and Results

The above scheme was tested using data collected on a Varian 600MHz Spectrometer with a 4-parallel RF coil array. A spin-echo (Tr=1s, Te=12ms) pulse sequence was used to image a slice through the head of a mouse, and the k-space sampling was optimized off-line for different acceleration factors. Since 2D Fourier imaging was used, only the density of phase-encodes needed to be optimized. The figures below show the 256×256 reconstructions obtained using (a) the full data-set with 256 phase-encodes acquired at Nyquist spacing, (b) the MMSE reconstruction from 64 optimally spaced phase encodes. (c-d) SENSE and regularized SENSE reconstructions from 64 phase encodes at 4-times Nyquist spacing.

Conclusions

As seen from the images the optimal variable density sampling can achieve higher acceleration factors without significant imaging artifacts. Regularizing the SENSE reconstruction can reduce, but not eliminate, the aliasing artifacts due to undersampling in k-space. The normalized rms reconstruction errors ($\|\mathbf{f} - \hat{\mathbf{f}}\| / \|\mathbf{f}\|$) for the images in Figures (b-d) are1.5%, 230%, 6% respectively.

Thus the proposed method can be used to optimize the k-space sampling locations in a parallel MR experiment, providing gains in SNR or reduction in imaging time. The method is applicable not only for 2D Fourier imaging, but also can be used for 3D imaging or to optimize variable density spiral imaging.



(a) (b)

(c) (d)

References

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