Magnetic Resonance Imaging of Coulomb Forces

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Introduction: Magnetic Resonance Elastography (MRE) is a progressively growing tool among the modern imaging techniques in medicine [1]. It utilizes a phase contrast MRI and mechanically generated shear waves (SW) to quantitatively measure the shear modulus of tissue. There are several ways to induce SW via external forces such as electromechanical drivers, pneumatic drivers, piezoelectric drivers focused ultrasound and others [2, 3]. Many of them rely on solid contact between the surface of a moving part of the driver and a surface of the medium. This surface force produces SW that originate at the surface and attenuated as they propagates through the material. The attenuation is governed by the elastic properties of the material and depends on the frequency and amplitude of the excitation force. Another way to produce displacements is to generate them by a body force. In this case we can avoid some of the wave attenuation. In a conductive material a body force can be induced by an alternating electromagnetic field (EF). A finite conductivity implies the presence of charged molecular complexes which tend to move under the influence of an EF. The magnitude of this motion is partly defined by the intensity and frequency of the applied EF. In a gel-like tissue the displacements of a charged complex produce a drift of the surrounding medium which is the function of the elastic properties of the material and can be detected by the means of MRE.

Methods: A rectangular 2% agar phantom[1] was subjected to the EF stress shown in Fig.1. The phantom was poured into a fiberglass frame with platinum electrodes on opposite sides of the phantom at X=0 and X=A. The phantom contained 0.9% of NaCl to match the conductivity similar to biological tissues. The dimensions along the X, Y and Z axis (A, B and C in Fig.1) were 9.9, 8.2 and 2 cm respectively. When alternating voltage V_0 is applied across the phantom (along the X axis), the longitudinal displacements caused by Coulomb forces have the same direction as the external EF. Since the length of the longitudinal electromagnetic wave can be estimated as $2x10^8$ m, the transverse (shear) wave can also be assumed to exist without interaction with the former. The equation for the transverse displacements U_y in the Y direction is:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)U_y-\frac{1}{C_t^2}\frac{\partial^2 U_y}{\partial t^2}=0\,, \quad (1)$$

where C_t is the velocity of the SW given by $(\mu / \rho)^{1/2}$. Here μ and ρ are the shear modulus and the density of the material. The general form of a solution of this equation by the method of separation of variables can be written as:

$$U_y(x,y,z,t)=P(x,y,z)\cdot e^{-i\omega t}=k(x)h(y)g(z)\cdot e^{-i\omega t}\,, \quad (2)$$

where *k(x)*, *h(y)* and *g(z)* are functions of position and $\omega = 2\pi f$ is the angular wave frequency. Solving (1) using (2) and the no-slip boundary conditions on the fiberglass walls and the force-free region on the upper surface, yields the expression for the eigenfrequencies of the system:

$$f=\frac{1}{T}=\frac{c_t}{2}\sqrt{\left(\frac{m}{A}\right)^2+\left(\frac{n}{B}\right)^2+\left(\frac{2l+1}{2C}\right)^2}\,. \quad (3)$$

Here *m*, *n* and *l* are equal to 0, 1, 2… and represent the number of half-wavelengths that fit into the dimensions of the phantom along the X, Y and Z axis respectively.

Results: Eq.(3) was evaluated to determine the eigenfrequencies for different resonance mode. For mode m = 2, n = 2, l =0 in the 2% agar gel phantom (μ = 23 KPa) the eigenfrequency is equal to 96.6 Hz. The experimental results are shown in Fig.2 and Fig.3. The alternative voltage of 97 Hz with the peak-to-peak amplitude of 51 Volts was applied across the phantom. Coronal 2D MRE data in the X-Y plane at Z = 1cm were collected at 8 time offsets between the motion and motion-sensitizing gradients with motion encoding along the X and Y axes. Fig.2 shows phase/wave images in degrees at one time offset for each sensitizing direction. Fig.3 represents the phase data along the white lines in Fig.2 for each of the eight time offsets: t = 0 and T (beige); T/8 (violet); T/4 (brown); 3/8 T (blue); T/2 (navy); 5/8 T (red); 3/4 T (green) and 7/8 T (navy-blue).

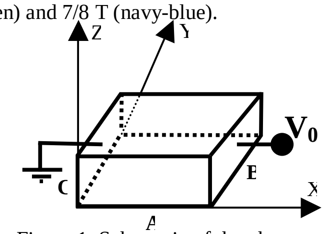

Figure 1. Schematic of the phantom. $A=9\,cm;\ B=8\,cm;\ C=2\,cm.$

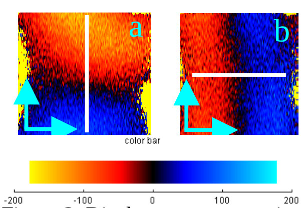

Figure 2. Displacement pattern in the coronal slice. a) X-sensitized motion; b) Y-sensitized motion.

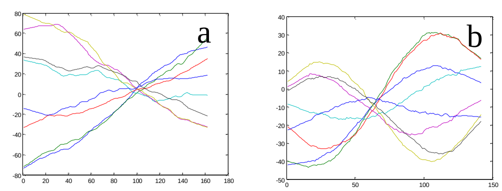

Figure 3. Phase along the lines shown in Fig.2. for each of the time offsets.

Discussion and Conclusions: The above data demonstrate the feasibility of MRE to measure displacements induced by body forces from EF in conductive materials. We have also found increasing of amplitude of the displacements at the prominent frequencies which are close to the calculated resonant frequencies. The resonance frequencies of the system can be predicted with the knowledge of materials, the geometry and the boundary conditions. In Eq.(3) we assumed fixed boundaries along the sides of the phantom. Fig.3b shows that the boundaries at X=0 and X=A (the electrodes) hold this condition while no-slipping boundary condition is violated at y = 0 and y = B, as Fig.3a indicates. With an accurate model of the geometry and the boundary conditions of the experiment, knowledge about the resonant frequencies can be used to determine elastic properties of the material.

References. [1] R. Muthupillai, D.J. Lomas, P.J. Rossman, J.F. Greenleaf, A. Manduca, R.L. Ehman. Science, 1995; 269:1854-1857.
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[1] This method can be extended to other regular shapes with specified boundary conditions.