A Simple 3D *k*-Space Trajectory Design Method for MRSI and MRI Applications

S. Uribe[1], R. Mir[1], A. Guesalaga[1], J. Spiniak[1], P. Irarrazaval[1]

[1]Electrical Engineering, Pontificia Universidad Catolica de Chile, Santiago, Chile

Introduction. In 3D MR spectroscopic imaging, MRSI *(x,y,z,f)*, and 3D MRI *(x,y,z)* faster *k*-space trajectories are needed to reduce the scan time. A common approach for fast 3D MRSI is to use echo planar (EP) trajectories. For 2D MRSI it has also been proposed to use spiral trajectories [1]. We propose to adapt the insights of several fast 3D MRI trajectory design strategies, such as Projection Reconstruction (PR), Stack of Spirals (SOS), "Twisted" Projection Imaging (TPI) and Missile Guidance (MG) [2,3,4], to design a fast 3D MRSI trajectory. Our design is simple and consists on rays departing from the origin in revolving spheres. This approach guarantees good image quality as well as short scan times compared with other fast trajectories. The method was tested in both 3D MRSI and MRI. It can also be extended for dynamic under-sampled imaging.

Method. The method of design is based on five key ideas: **(1)** *N* points representing the number of required shots, are uniformly distributed over the surface of a sphere of infinitesimally small radius [5]. They also define the initial azimuthal and polar departing angles of the trajectory: ϕ_0 and θ_0. The relative angular position of these points is constant for every shell (independent of the radius). **(2)** The radial velocity is defined by *Equation 1* in order to get uniform distribution in radial directions [2,3]. **(3)** As the radius increases, the sphere is simultaneously rotated around two axis as shown by *Equation 2*, where $\mathbf{M_y}$ and $\mathbf{M_z}$ are rotation matrices. This maximizes the *k*-space coverage, as the radial velocity is increasingly restricted. **(4)** The angular velocity of the sphere, $\boldsymbol{\omega}(t)=[\omega_1(t) \quad \omega_2(t)]$ and the angular acceleration, $\boldsymbol{\alpha}=d\boldsymbol{\omega}/dt$, must be equal for every shot and defined by *Equation 3* to comply with hardware limitations (G_{max}, SR_{max}). In order to get an efficient *k*-space coverage, at least one of the shots is forced to use the maximum gradient and both components of $\boldsymbol{\omega}(t)$ are optimized resulting in $\omega_1/\omega_2=1.5$. **(5)** *k*-space is divided into small cubes, with sides defined by the Nyquist criterium; hence the coverage percentage is proportional to the quantity of cubes containing at least one sample.

$$\dot{k}_r = \frac{K_{\max}^{\ 3}}{3 \cdot T_{Acq} \cdot k_r^{\ 2}}$$

(Where k_r is the radial frequency position, T_{Acq} the acquisition time and K_{max} the sphere radius)

Equation 1

$$\begin{bmatrix} \dot{k}x(t) \\ \dot{k}y(t) \\ \dot{k}z(t) \\ \dot{k}f(t) \end{bmatrix} = \begin{bmatrix} & \mathbf{M}_y(\omega_1 t)\mathbf{M}_z(\omega_2 t) & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dot{k}_r(t)\sin\phi_0\cos\theta_0 \\ \dot{k}_r(t)\sin\phi_0\sin\theta_0 \\ \dot{k}_r(t)\cos\phi_0 \\ 1 \end{bmatrix}$$

Equation 2

$$\dot{k}(t) = \dot{k}_r(t)\mathbf{e_r} + \boldsymbol{\omega} \times kr(t)\mathbf{e_r} \le \gamma \cdot \left\| G_{max} \right\|$$

$$\ddot{k}(t) = \dot{k}_r(t)\mathbf{e_r} + \boldsymbol{\alpha} \times kr(t)\mathbf{e_r} \le \gamma \cdot \left\| SR_{max} \right\|$$

Equation 3

Results and discussion. The proposed method was tested via computer simulation in a 3D MRSI experiment and also in MRI under non-ideal condition. The design parameters were: FOV *10cm x 10cm x 10cm x 3.8kHz* (*16 x 16 x 16 x 307 or 6.25mm x 6.25mm x 6.25mm x 0.6 ppm at 0.5 T*) for MRSI and FOV *4,4cm x 4,4cm x 4,4cm (44x44x44)* for MRI. A gridding algorithm with a triangular kernel was used for reconstruction (the pre-density was analytically estimated). An example of the propose trajectory is shown in *Fig 1* (only five waveforms are shown). For spectroscopic imaging simulations, we used an object formed by a cube and a sphere with different resonant frequencies. The analytical phantom was sampled with the proposed trajectory and reconstructed. *Fig 2* shows two slices of the 3D images at each frequency. For 3D MRI a unique characteristic of this method is the uniform distribution of points in every shell. Therefore it is possible to undersample *k*-space for the same acquisition time than SOS or MG, but with better imaging quality under non-ideal conditions. MG and the proposed trajectory show similar and good behavior. SOS deteriorates considerably as shown in *Fig 3*. Another advantage of the method is that *k*-space is sampled progressively. In other words the sampling is split into complementary groups of shots, where the contribution of each one increases the coverage, so intermediate stages maintain an even surface density for a given radius. This means that if the patient moves during the exam, the experiment can be aborted and useful images can still be obtained; something hardly achievable with other 3D trajectories.

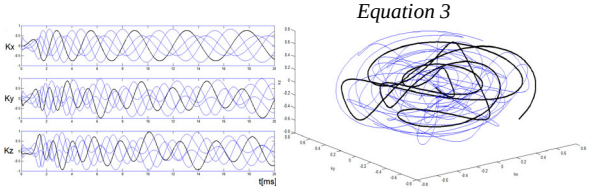


Fig. 1. Gradient waveforms and shots for 3D k-space trajectories. One shot is bolded for better visualization.

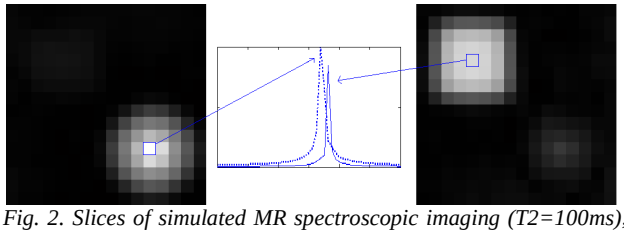
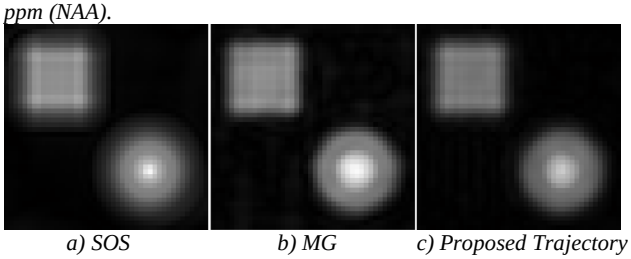


Fig. 2. Slices of simulated MR spectroscopic imaging (T2=100ms), frequency sphere = 3.03 ppm, (Creatine), frequency square = 2.02 ppm (NAA).



a) SOS *b) MG* *c) Proposed Trajectory*

Fig. 3. Slices of simulated MR imaging under non-ideal condition (2.5 ppm constant off-resonance and T2 =100 ms) for equal scanning times.

Conclusion. In this paper we propose a simple method for 3D *k*-space trajectory design. For MRSI this trajectory can be utilized to recognize different compounds in short scanning times and in MRI good image quality can be obtained even under non-ideal conditions.

References. [1] E. Alsteinsson et al, *MRM. 36 889-98 (1998)*; [2] P. Irarrazaval et al, *MRM. 33, 656-662 (1992)*; [3] F. Boada et al, *MRM 37, 706-715 (1997);* [4] R. Mir et al, *MRM. 52 329-336 (2004).* [5] S. Wong et al, *MRM. 32 778-784 (1994).*