

# Spatial-Spectral Holographic Interpretation of the MR Imaging Problem: Analysis and Implications

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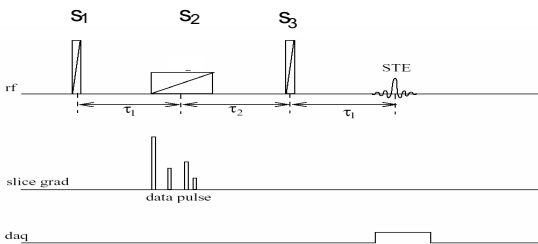
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## Abstract:

The fundamental MR imaging equation can be derived from a generalized spatial-spectral holographic wavefront reconstruction interpretation similar to that in quantum optics. A spatial-spectral holographic interpretation arises naturally in MR from an inhomogeneous linewidth broadening ( $\sim 1/\pi T^*$ ) due to either an imposed set of linear orthogonal gradients fields or from the intrinsic chemical shift/molecular environment surrounding the spins or from the multi-nuclear spin system of the sample. We can thus think of the imaging sample (in the inverse image space or k-space) as a spatial-spectral holographic grating with the RF excitation acting as the 'read-out' beam that is spectrally 'Bragg-matched' to a slice or slab of the spatial-spectral grating  $M(k_x, k_y, k_z, k_f)$ . In this work, it is shown analytically and experimentally that an inhomogeneously broadened NMR absorber can accomplish key holographic characteristics such as storage, recall, time-reversal and matched filtering of gradient weighted RF pulses in a three pulse stimulated echo experiment.

## Methods

The MR imaging equation can be interpreted as a 3D diffraction off a spatial-spectral grating such that the imaging volume represents an inhomogeneously broadened RF absorber in direct analogy to optical coherent transients. An RF excitation, coincident with an applied gradient field acts as a 'read-out' pulse with a spatial-spectral bandwidth that is 'Bragg-matched' to the grating vector  $M(k_x, k_y, k_z, k_f)$  which thus diffracts into signals whose inverse Fourier transform is the desired image  $m(x, y, z, \sigma)$  projected along the chemical shift axis  $\sigma$ , for each MR active chemical species in the sample. Readout then represents a 'burning' of a spectral hole in the absorption spectrum of the spin resonant system. The gradients thus play the additional task of spectrally multiplexing the hologram read-out via slice selection and frequency/phase encoding. The requirement is to sample this projection space (k-space) adequately so as to reconstruct the object with negligible distortion.



$s_1(t)$	$s_2(t)$	$s_3(t)$	$o(t)$	Application
				Programmable time delay
				Data Storage
				Time-reversal
				Correlator
				Triple Product

Figure 1: Timing diagram for storage, recall and filtering of RF pulses in an MR sample. The data pulses are applied as a slice selective gradient so that in combination with the chirped RF pulse  $s_2$  result in a spectral hole in the medium. This data dependent spectral hole distribution is interfered with the population excitation of the first chirped RF pulse  $s_1$  to create a spatial-spectral grating. The table shows permutations of potential processing functions from a combination of these three pulse sequence. Data processing alternatives in an gradient induced inhomogeneity in an NMR sample based on a three pulse stimulated echo experiment. For example, when the 1<sup>st</sup> and 3<sup>rd</sup> pulses are very short then the emitted signal is a temporal replica of the 2<sup>nd</sup> pulse –storage. Without loss of generality, data pulses are shown as chirps but any crafted pulses can be used as long as their spectral content is within the inhomogeneous bandwidth of the medium.

## Results:

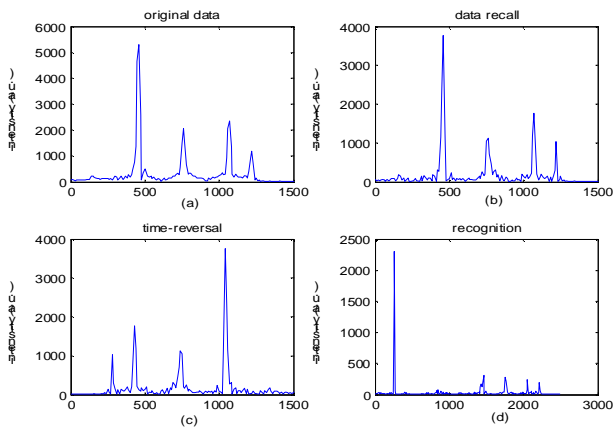


Figure 2: Storage, recall and matched filtering of a pulse consisting of four sub pulses in a three pulse stimulated echo experiment with an MR phantom. Data consisting of four sub-pulses that are used to study the storage and recall properties of MR holography. The reference pulse is applied to the RF transmitter channel while the data channel is applied to one of the gradient directions. In general, either the RF or the gradient channels can be used to carry either the reference or the data pulses. During the recording of the spatial-spectral grating, two pulses consisting of the reference ( $S_1$ ) and the signal ( $S_2$ ) spectrally interfere in the inhomogeneously broadened NMR absorber. (a) shows the original signal (data) pulses; (b) if the reference pulse is applied first, then the free induction decay signal has the same time order as the signal pulse (storage) (c) if the signal pulse is applied before the reference pulse, then the signal pulse echoed from the spatial-spectral grating is time reversed (conjugate holography) (d) cross-correlation function is obtained from temporal matched filtering.

## Conclusions:

Fundamental properties of holography such as storage, recall and matched filtering have been experimentally demonstrated. This holographic interpretation of MR has implications on alternative spatial encoding schemes, new approaches to field homogeneity shimming using phase conjugate imaging principles as well as imaging using controlled gradients induced quantum beats in optical coherent transient materials.

## References:

[1] P. Mansfield and P. K. Grannell "NMR diffraction in Solids," J Phys C Solid State 6, 422-426 (1973).