Mixed LS-TLS method for optimal SENSE reconstruction

P. Qu[1], J. Yuan[1], J. Wei[1], G. X. Shen[1]

[1]Dept. of Electrical and Electronic Engineering, The University of HongKong, HongKong, HongKong, Hong Kong

Introduction

In SENSE [1] reconstruction, the conventional least squares (LS) solution is not optimal in presence of sensitivity noises; instead, the total least squares (TLS) method has proven effective when sensitivities can not be accurately estimated [2]. However, the TLS formulation assumes that the errors in sensitivity matrix are independently and identically distributed with the same variance, which is usually not true in practice. For example, if coil sensitivities are calibrated by division of single-coil images by a reference image, SNR distribution in sensitivity maps will be non-uniform and high noise level arises in the regions of low image intensity. This means that in the sensitivity matrix to be inverted, some columns are accurately known and others are error-prone. To find the optimal estimation, in this study the unfolding process is formulated as a mixed LS-TLS problem [3]. Considering the credibility information of sensitivity matrix, this novel approach can lead to more accurate reconstruction than both LS and TLS solutions.

Method

The unfolding process of SENSE involves a LS solution of a linear system of equations (LSE): $Ax=b$, where A is the whitened sensitivity matrix which takes the receiver noise covariance into account [1, 4]. If A is highly noisy, the TLS solution has proven a more suitable formulation which can be represented as the minimizer: $min||(E,r)||_F, \quad (A+E)x=b+r$, where $|| \bullet ||_F$ denotes the Frobenius matrix norm.

In our approach, the sensitivity matrix is divided into two blocks according to their different credibility and a mixed LS-TLS solution can be found using tools of linear algebra. The mixed LS-TLS problem can be expressed as: $min||(E,r)||_F, \quad (A_1, A_2+E)x=b+r$, (1)

where A_1 is formed by the columns of sensitivity values which can be considered as accurate, and A_2 consists of noisy sensitivity values. Here we assume that the noise distribution in sensitivity maps is a *priori* information.

Solution of Eq. (1) can be obtained by the following procedures [3]:

(i). QR decomposition: $(A_1, A_2, b) = Q\begin{pmatrix} R \\ 0 \end{pmatrix}, \quad R = \begin{pmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{pmatrix}.$ where A_1 is $m \times n_1$, A_2 is $m \times n_2$, and R_{22} is $(n_2+1)\times(n_2+1)$ matrix.

(ii). Compute SVD: $R_{22} = U\Sigma V^T, \quad \Sigma = diag(\sigma_1, \sigma_2, ..., \sigma_{n_2+1})$

(iii). Determine $k \le n_2$ such that $\sigma_k > \sigma_{k+1} > .... > \sigma_{n_2+1}$. Set $V_{22} = (v_{k+1}, \ldots v_{n_2+1})$ and compute V_2 by: $R_{11}V_{12} = -R_{12}V_{22}, \quad V_2 = [V_{11}; V_{12}]$.

(iv). Perform Householder transformation: $V_2 Q = \begin{pmatrix} Y & Z \\ 0 & t \end{pmatrix}$ and then the solution can be obtained by: $x = -Z/t$.

Results

In vivo MRI experiments were performed on a healthy volunteer on a Siemens 1.5T NOVUS scanner equipped with four independent receiver channels. Abdominal MRI data were acquired with a coil array consisting of two anterior elements from a flexible torso coil array and two posterior elements chosen from a planar spine array. For this experiment, a full dataset was acquired and later decimated off-line by a factor of 2 to simulate *2X* acceleration. A reference image was reconstructed by FFT of the full dataset and “sum-of-squares” combination. Sensitivity maps were estimated by conventional “quotient of images” method. The *2X* SENSE reconstruction results using LS solution, TLS solution and mixed LS-TLS solution are shown in Fig. 1a, Fig. 1b and Fig. 1c, respectively. To compare their SNR more clearly, the difference between the three reconstruction results and the reference image are displayed in Fig. 1d, Fig. 1e, and Fig. 1f, respectively.

The results show that in presence of non-uniformly distributed sensitivity noises, the mixed LS-TLS solution leads to better SNR than both LS and TLS solutions. In general, the mixed LS-TLS method can accommodate different sensitivity noise distributions and thus it is the most useful method in most cases.

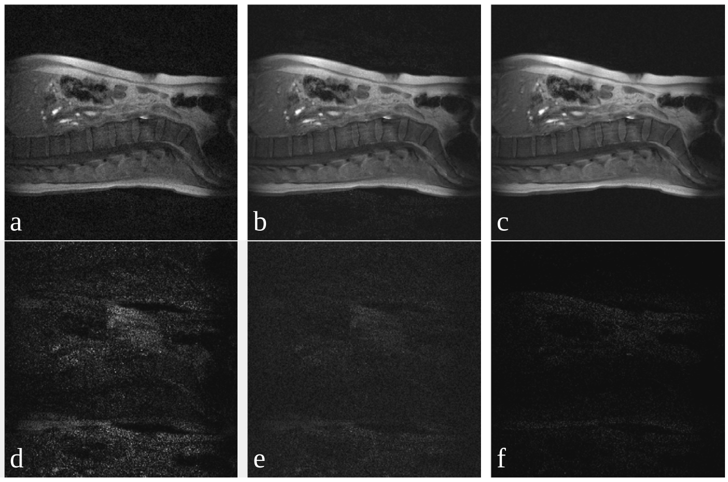

Fig. 1. SENSE reconstruction results of an abdominal image: (a). using LS solution; (b). using TLS solution; (c). using mixed LS-TLS solution; (d-f). The difference between (a), (b), (c) and the reference image, respectively.

Conclusion

In parallel imaging, since the noise distributions in sensitivity maps are usually non-uniform, neither the LS solution nor TLS solution is optimal to invert sensitivity matrix. In this study a mixed LS-TLS method has been proposed for optimal SENSE reconstruction. This hybrid method takes the spatial distribution of sensitivity noises into account and it has proven more effective than both LS and TLS methods.

Acknowledgement

This work was supported by RGC Grant 7045/01E and 7170/03E.

References

[1] K.P. Pruessmann, et al. MRM 42: 952-962 (1999)
[2] Raj A. et al. 12th ISMRM proceedings, p. 2411, 2004.
[3] G. Dahlquist, *Numerical Mathematics and Scientific Computation*, SIAM press, 2003
[4] F.H. Lin, et al. MRM 51:559-567(2004)