Adaptive Phase-Constrained Reconstruction For Partial Fourier Partially Parallel Imaging

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Introduction: Over the years, exploring redundancy of MRI data has been one of the major strategies to achieve faster scanning. Partial Fourier (PF) techniques (1) explore the redundancy of MRI data stemming from the Hermitian symmetry of Fourier transform of a real-valued function. Partially parallel imaging (PPI) techniques (2) use the redundancy of multicoil data arising from the differences in coil sensitivities. PPI can be utilized with PF in several ways, including PPI followed by PF reconstruction (3) or combining them in a single procedure (4-6). Simultaneous treatment of PF and PPI was shown to suppress noise amplification (4). Application of other types of phase constraints (7) is also possible to provide noise suppression (4).

Applying phase constraints in both PF and general phase constrained PPI requires a reliable estimate of the image phase, which is obtained *in situ* from a low-resolution scan. Errors may be introduced into image magnitude if the image phase deviates significantly from its low-resolution estimate. In PF MRI, the errors often exhibit themselves as local degradation of image resolution. In phase constrained PPI such as PF-PPI, the phase errors become even more malignant, as they are not localized any more and become spread in the image plane. This prohibits application of PF methods in situations when rapid phase changes occur. As described in (6), one way to reduce these artifacts is to vary the dependence of the reconstruction on phase constraint using a regularization procedure:

$$\begin{bmatrix} \mathrm{Re}\{\mathbf{s}\} \\ \mathrm{Im}\{\mathbf{s}\} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\{\mathbf{B}\} & \mathrm{Im}\{-\mathbf{B}\} \\ \mathrm{Im}\{\mathbf{B}\} & \mathrm{Re}\{\mathbf{B}\} \\ \mathbf{0} & \lambda\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathrm{Re}\{\boldsymbol{\rho}\} \\ \mathrm{Im}\{\boldsymbol{\rho}\} \end{bmatrix} \qquad [1]$$

where **B** is a sensitivity matrix, $\boldsymbol{\rho}$ is a vector of image pixels to be reconstructed, **s** is a vector of acquired k-space samples, λ is a scalar and **I** is the identity matrix. The "regularization" parameter λ allows a smooth transition between unconstrained ($\lambda = 0$) and fully constrained ($\lambda \to \infty$) reconstruction.

One weakness of the above method is that it may under-utilize the phase constraint in areas where phase is well represented by low-resolution estimate, and over-utilize it in the areas of rapid phase changes. An alternative approach is to apply the phase constraint adaptively by specifying a value of λ for each individual pixel. This allows the constraint to be weak in those places where the phase varies rapidly – thus avoiding artifacts – and strong everywhere else to minimize noise amplification. The way this is done is by replacing the $\lambda\mathbf{I}$ term in Eq [1] with a diagonal matrix **L** and choosing the elements of **L** to reflect the local phase properties in the image. This type of numerical processing is based on ideas from "ridge regression".

Method: We use a two-stage procedure to demonstrate the proposed concept. On the first pass, an unconstrained reconstruction is performed to gain a full-resolution estimate of the underlying phase (albeit contaminated with noise). This is used to guide the second pass reconstruction, in which the phase constraint is applied using a spatially varying constraint via the matrix **L**. Among other possible choices, the elements of **L** are chosen on the following basis:

$$\mathbf{L} = \frac{1}{\left(1 + \left| \angle\boldsymbol{\rho} \right|\right)^2} \qquad [2]$$

where $\angle$ denotes the angle in radians and $\boldsymbol{\rho}$ is the image produced by the first pass reconstruction. The precise form of Eq 2 does not seem to be especially critical – as long as it returns a small value when $|\angle\boldsymbol{\rho}|$ is large, and a large value otherwise – presently it is not known whether there may be a "best" function for this purpose.

Results: The approach was validated on simulated data using BrainWeb digital brain phantom (7). Coil sensitivities were modeled as 2D Gaussian functions for magnitude part and 2D linear functions for phase part.

A spin-echo data set was acquired: 8 coils, 256×256 resolution and an auto-calibrating k-space sampling strategy (48 central lines, outer-line spacing 5, 90 lines in total). Reconstructions were performed in MATLAB. The results are shown in Fig. 1 and Fig. 2.

Discussion: The advantage of the new adaptive technique for phase constrained PPI reconstruction is its ability to determine the appropriate phase constraint for each pixel in the image, which results in reduced artifacts and/or better SNR. We anticipate that such approach may be useful for reconstruction of phase contrast MRA data, where rapid phase changes in the vessel areas make application of PF techniques problematic. Further research is required to estimate the potential of the new technique for PF / PPI phase contrast MRA.

The main disadvantage is reconstruction time, which is doubled by using a two-stage process. There is also some uncertainty as to what the best way of choosing **L** might be although this does not appear to be critical to the quality of the final image. The optimal expression for **L** is under investigation. Possible extensions of the proposed algorithm include applying postprocessing to the phase after the first stage to minimize the impact of noise on values in **L**.

References: (1) Liang et al. Rev MRM 1992;4:67 (2) Pruessmann et al. MRM 1999; 42:952 (3) King et al. ISMRM 2000:153 (4) Samsonov et al. MRM 2004;52:1397 (5) Willig-Onwuachi et al. ISMRM 2003:19 (6) Bydder et al. ISMRM 2004:532 (7) Samsonov AA et al. ISMRM 2002:2408 (8) Collins DL et al. IEEE TMI, 199817:463-468.

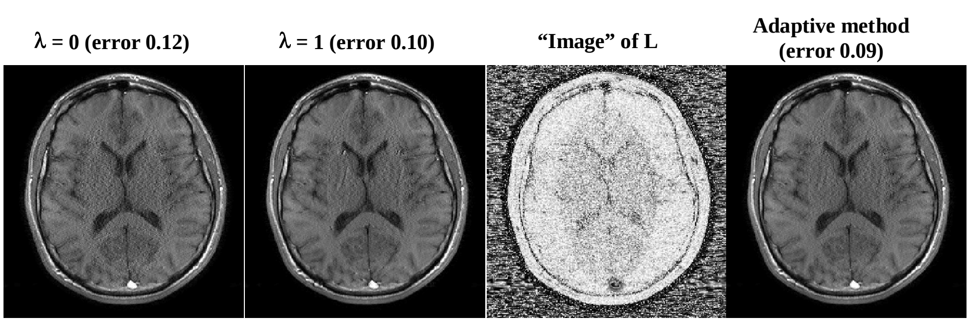

Figure 1. Adaptive phase-constrained reconstruction of PF PPI data (N_C=8, R=5).

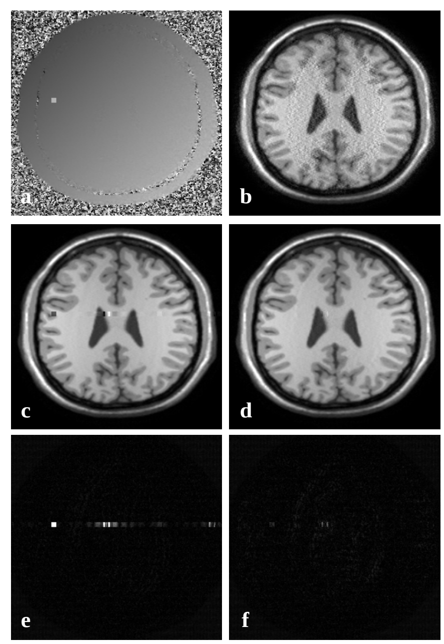

Figure 2. Simulated data studies (N_C=4, R=3, regular Cartesian undersampling, no PF). **a:** Phase map of a single coil with phase error (small square). **b:** Unconstrained reconstruction. **c:** Nonadaptive phase-constrained reconstruction. **d:** Adaptive phase-constrained reconstruction. **e,f:** Error differences corresponding to *(c,d)*.