Generalised g-factor Reduction Using Joint Entropy

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Introduction: Partially Parallel Imaging (PPI) can reduce the number of phase encoding steps required when using array coils, resulting in speedup, but also leads to noise amplification in the final image. Such noise amplification (g-factor effect[1]) is recognised as a limiting factor for the maximum speed-up that can be achieved in practice[2]. Previous work by the authors[3] demonstrated the use of the principal eigenvector of the inverse sensitivity matrix as a means for determining the distribution of noise in the final images which could them be minimised subject to minimising joint entropy, E_j ,[4] between the target image and a reference. This approach was limited to the fully determined case. G-factor noise can still be a problem for the over-determined case, i.e. when the speed-up factor is less than the number of coils, and here we present the generalisation of the method to any speedup factor.

Theory: For SENSE with regular undersampling, aliased pixels are separated using linear algebra into regularly spaced families of final image pixels. If the corresponding aliased pixels for each coil are assembled in to a complex vector ***S*** and the unfolded pixels are written as a complex vector ***X***, then: $X=C^{-1}S$ where ***C*** is a (complex) reconstruction matrix that is derived from the coil sensitivities. In practice noise is present in any measurement. Here we consider only uncorrelated noise in the target images ***S***, then we have $X' + \partial x = C^{-1} \cdot [S' + \partial S]$ where *X'* is an ideal noise free solution, *dx* is the g-factor enhanced noise, ***dS*** is a noise vector and ***S***' is an ideal noise free measured signal. From this we can write: $\partial X = C^{-1} \cdot \partial S$ This provides a recipe for the distribution of noise across the final images given knowledge of the input noise in each coil. However the input noise is not known and is a vector with length equal to the number of coils. An unconstrained solution would require determining a free parameter for every coil for each pixel family. To constrain the problem we note that when the g-factor is high, C is ill conditioned and there tends to be a single dominant eigenvector. For the exactly determined case we can now approximate the elements δX by $\partial X_{(Ceig)} = \partial s.\alpha.\lambda$, where λ is the principal eigenvector of C^{-1}, α its eigenvalue and δs is a single unknown complex scalar that is to be determined. In reference 3 δs was determined by minimising the joint entropy (E_j) between the SENSE reconstructed image with δx (as estimated from δs) subtracted off, and a reference image that does not suffer g-factor noise. The reference may have the same or different contrast and/or resolution. The result is a solution constrained by the noise correlations within the family of pixels as parameterised by δs. This constraint gets stronger the more pixels are linked in the image (the higher the speed up factor). Under these conditions the g-factor is usually worse also. The general solution for the over determined case (where the speed up factor is less than the number of coils) invokes the linear least squares formalism and replaces the principal eigenvector of the inverse sensitivity matrix directly with the principal eigenvector of the variance-covariance matrix *Q* where: $Q=(C^tWC)^{-1}$, superscript t indicates transpose and W is a square array containing confidence estimates for the elements of C. When all elements of C have equal confidence W is set to a diagonal unitary matrix and $\partial X_{(Qeig)} \approx \partial s.\sqrt{\alpha_Q}.\lambda_Q$, where α_Q is the principal eigenvalue and λ_Q the principal eigenvector of *Q*. An approximation for the g-factor noise free object can written: $X' = \left[C^{-1} \cdot S\right] - \partial X_{(Qeig)}$. Minimisation of E_J [4] between X' and a reference image is then used to determine the optimal image.

Method: The method was tested using synthetic images from the MNI brain phantom[5] and images from a 3T Philips scanner using a 6 channel head coil. The synthetic images were multiplied by candidate coil profiles before being transformed to k-space, sub-sampled and then reconstructed again as aliased images. Independent complex Gaussian noise was added to each aliased coil image and a SENSE reconstruction performed. Tests were performed with a linear speed-up factor of 6, 5 and 4 using a model linear array of 6 coils. In the examples shown (speed up 5) the reference image for the entropy calculation was chosen to have the same resolution as the target images but did not have synthetic MS lesions preent. Also shown in figure 1 is real data taken from a 6 channel head coil on a Philips 3T scanner at a speed up factor of 5.

Results: The method resulted in reduction of g-factor noise for all speedup factors to a level similar to the surrounding areas or reduced pixel degeneracy. The principal eigenvector approach is the best approximation for pixels with degeneracy equal to the speed up factor, where the shape of the object being imaged dictates that the pixel degeneracy is less than this then the correction is less good, however in these areas the noise is also less amplified.

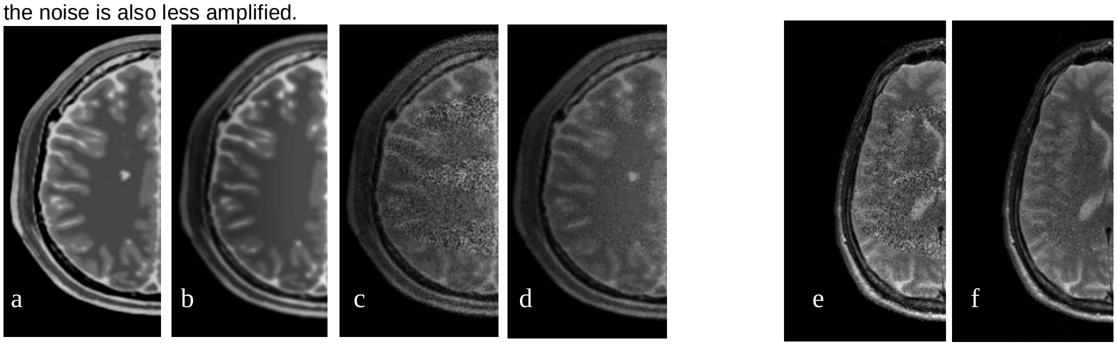

Figure 16 coil speed up factor 5 a) Gold standard image b) image used as reference for the entropy optimisation (note lack of lesion present in real data). C) start data where lesion is almost entirely obscured by g-factor noise. d) final image revealing lesion. E) real data from philips 3T system. F) corrected image.

Discussion: The method now reduces g-factor noise at any speedup. In these examples highly comparable reference images were used, the effects of reduced resolution and different contrast images for the entropy reference is being investigated within this new general formalism. Processing time is long due to the search nature of the optimisation (see reference 3) however a multi scale approach has reduced computation time by an order of magnitude to 3 hrs per image and we expect to achieve further significant reduction.

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References:[1]Pruessman et al Magn Reson Med 1999 Nov;42(5):952-62. [2]Wiesinger et al ISMRM 2002 p191.
[3]Larkman et al ISMRM 2004 Kyoto p329 [4]Shannon et al Bell Syst Tech J vol 27 pp379 1948. [5]http://www.bic.mni.mcgill.ca/brainweb/.