Improved SENSE reconstruction by pre-scaled TLS method

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Introduction

In parallel MRI techniques such as SENSE [1], the unfolding process usually involves solving an over-determined linear system of equations (LSE) in the form $Ax=b$, particularly in SENSE A is the sensitivity matrix containing the complex sensitivity values of each receivers at superimposed pixels. Conventionally this LSE is solved using a least squares (LS) estimation, i.e., $x = pinv(\bar{A})*b$, where *pinv* denotes *Penrose-Moore* pseudoinverse and $\bar{A}$ is the whitened sensitivity matrix which takes the receiver noise covariance into account [1, 2]. In some cases coil sensitivities can not be accurately estimated and consequently matrix A is significantly affected by sampling errors. In this study an errors-in-variables model is employed in which the LSE is pre-scaled and solved by the total least squares (TLS) method [3]. By carefully choosing the relative scaling parameter for the LSE, the pre-scaled TLS method leads to more accurate solution than conventional LS method when sensitivity maps are not accurately estimated. Hence the reconstructed images have less aliasing artifacts and better SNR.

Method

The ordinary LS problem is to find a solution x for the LSE $Ax = b$ to minimize the Euclidian length of the residual vector, i.e., find the minimal perturbation in b to make the LSE consistent: $min||r||_2$, $Ax = b+r$.

When the coefficient matrix A is not exactly known, for example, sometimes in SENSE the sensitivity matrix can hardly be accurately estimated, the errors-in-variables model is more suitable to find the optimal solution. In this model one assume a perturbation both in and A in b so that $(A+E)x = b+r$. The estimates of the unknown parameters x are obtained from the solution of the TLS problem: $min||(E,r)||_F$, $(A+E)x=b+r$, where $|| \bullet ||_F$ denotes the Frobenius matrix norm. The TLS solution significantly depends on the relative scaling of A and b. If we scale x and b by a factor α, the TLS problem is transformed into: $min||(E, \alpha r)||_F$, $(A+E)x=b+r$. When $\alpha \rightarrow \infty$ perturbations in b will be favored, which leads to the ordinary LS solution.

From linear algebra theory, the TLS problem can be analyzed using singular value decomposition (SVD) [3].

$(b,A)=U\Sigma V^T$, where $\Sigma= diag(\sigma_1, \sigma_2,\dots \sigma_{n+1})$ is the diagonal matrix containing singular values, $U=(u_1,u_2,\dots u_{n+1})$ and $V=(v_1,v_2,\dots v_{n+1})$ are the left and right singular vectors, respectively. The TLS solution can then be given by: $x = (v_{(2,n+1)},v_{(3, n+1)},\dots v_{(n+1,n+1)})^T / v_{(1,n+1)}$.

In this work the TLS method described above is implemented in a simulated SENSE reconstruction. Different scaling factors α are used in the unfolding process with variable noise levels in sensitivity maps. The results are compared in terms of SNR.

Results

The sensitivity profiles of a circularly arranged 8-element head coil array were simulated using an analytic integration of Biot-Savart equation for an axial FOV. Then these coil sensitivities were used with a standard Shepp-Logan phantom image to generate a full version of k-space data set. Simulated raw data with *4X* accelerations were then created by extracting partial dataset from the full data. During unfolding process, the sensitivity maps were artificially contaminated by variable levels of random noises. Different α were used to pre-scale the LSE, and then TLS method was used to estimate the optimal solutions. The normalized SNR of the reconstructed images varies with different choosing of the parameter α, as shown in Fig. 1. The curves show that in this particular reconstruction, when noise level in sensitivity maps (NLS) > -20dB, the TLS solution with α = 100~1000 leads to better SNR than ordinary LS method which corresponds to $\alpha=\infty$. The optimal scaling factor α for TLS reconstruction depends on the noise level in sensitivity maps. When noise level is very low, α should be very large and the optimal TLS solution is approximately the same with ordinary LS solution. However, when the sensitivity maps are not accurately estimated, TLS has better performance than LS and carefully choosing of α is necessary.

Fig. 2 shows the simulated *4X* SENSE reconstruction results (NLS = -15dB) using ordinary LS method (a) and TLS method with $\alpha = 100$ (b), respectively. It is visible from the images that the TLS solution leads to better SNR than LS method.

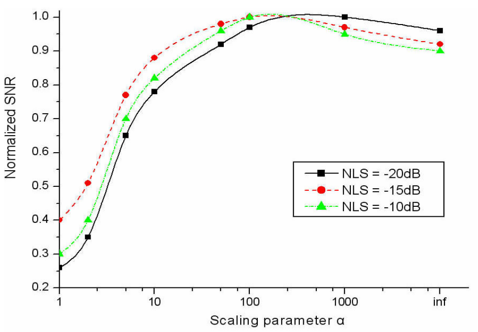
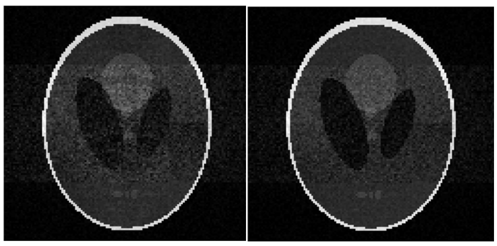


Fig. 1. Normalized SNR vs. scaling parameter for SENSE reconstruction with different noise level in sensitivities using TLS method



(a) (b)

Fig. 2. Simulated SENSE reconstruction results with noisy sensitivity maps (NLS= -15dB) using (a) ordinary LS method; (b) TLS method with $\alpha = 100$

Conclusion

Starting from the LS problem in SENSE, the pre-scaled TLS method has been introduced and its feasibility for inverting highly noisy sensitivity matrix has been verified by simulation. When sensitivity maps can not be accurately estimated, the TLS solution for the unfolding process with carefully pre-scaling has better performance than ordinary LS method.

Acknowledgement

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References

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