Augmented SENSE Reconstruction by an Improved Regularization Approach

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INTRODUCTION. Image reconstruction from undersampled k-space data acquired with parallel receivers, such as SENSE, can severely suffer from a poor conditioning of the design matrix. One major determinant for the conditioning of the design matrix is the geometrical properties of the receiver coil elements relative to size and location of the desired FOV and the degree of aliasing. Poor conditioning leads to residual aliasing and noise amplification in the final SENSE reconstruction. Ill-conditioned systems can be solved by regularization methods in which the solution stabilized by including appropriate additional information. Recently, SENSE reconstruction has become the focus for a variety of regularization techniques (1-4). In its original implementation, Cartesian SENSE reconstruction is typically done by traversing the aliased image and separating the aliased based on the spatial coil sensitivity characteristics of individual receive coils (5). For each set of n_a aliased pixels, the SENSE problem can be cast into an $n_c \times n_a$ matrix system (n_c is the number of coils), which is generally very small. Regularizing the solution vector for this inverse problem is limited by the paucity of singular values, which in case of truncated SVD regularization imposes harsh filtering of the design matrix. Moreover, the solution vector is usually comprised of pixels from distant and therefore unrelated positions and does not allow advanced penalty terms to take advantage of the continuous physical nature of the system. In this work, we present a more physically meaningful regularization method in which an entire line of pixels is unfolded at once using a single matrix inversion so that the solution vector represents an image profile. Therefore, penalty terms that impose some level of continuity on the solution vector, i.e. first and second derivatives, can be employed. Moreover, a much smoother damping of singular values is possible. Both computer simulations and in vivo experiments were performed to allow a comparative evaluation of the performance of different regularization schemes.

MATERIALS AND METHODS. *Numerical Phantom.* Sensitivity maps for 6 circularly-arranged coils were made with a Biot-Savart calculation and were used to modulate a Shepp-Logan phantom, on which Rice-Nakagami noise (SNR=50) was added thereafter and phase-encoding lines were eliminated (along y-direction) to achieve desired reduction factors from 2-6. To illustrate the effects of using regularization with prior knowledge (regularization image) (2) and assess potential pitfalls, an additional reference image was generated from the original phantom by blurring with a 2-pixel kernel, shifting by 1 pixel in the x-direction, and adding one anterior feature (ellipse). *In Vivo Data.* T2w-FSE images (256^2) were acquired with full k-space acquisition from a human volunteer with a GE Signa 1.5T scanner using an 8-channel head array (MRI Devices). A 3D FGRE (32×128^2) scan was performed for the coil sensitivity estimates. Desired aliasing was again achieved by eliminating the corresponding number of phase-encode lines. *Regularized Reconstructions.* Four methods of SENSE reconstruction were used: (i) the pixel-wise method typically used for cartesian data (5): at each pixel location in the undersampled data, **S**, a vector of n_a pixels, **x**, in the unfolded image, **U**, is found by solving a system of linear equations, **Ax**=**b**, using a least-squares method where the vector **b** is formed from the pixels of the individual coil components in **S**, and **A** is formed from the coil sensitivity estimates; (ii) similar to (i) but using Tikhonov regularization ($\|\mathbf{Ax}-\mathbf{b}\|_2 + \lambda \|\mathbf{L}(\mathbf{x}-\mathbf{x_0})\|_2 \rightarrow min$, $\mathbf{L}=\mathbf{I}$, $\mathbf{x_0}=0$) with L-curve analysis to determine the optimum regularization parameter λ; (iii) similar to (ii) but includes prior information ($\mathbf{x_0}$) from a reference scan (2); and (iv) performs column-wise unfolding, described as follows. For each pixel in a column (fixed frequency-encoding index) of **S**, the matrix **A** and vector **b** are formed, then a block-diagonal matrix $\mathbf{D}=\mathrm{diag}(\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_n)$ is made, where n denotes the number of rows in **S**, and a corresponding composite vector **c** is formed, $\mathbf{c}=(\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n)$. The solution vector **x**, with length equal to the number of phase-encodes (rows) in **U**, of the augmented system **Dx**=**c** now gives an entire column in **U**. However, the index mapping from **x** to the **U** is not monotonic; i.e., to get **x** to represent a profile in **U**, the columns in **D** need to be resorted. An example of the typical structure of **D** after sorting is shown in fig 1. This step is essential for using regularization methods (e.g., derivatives) that assume a relatively continuous set of values in the solution vector. For this column-wise method, we used a damped-SVD algorithm with the first-order derivative as the penalty term, **L**, and generalized cross-validation to find the optimum regularization parameter.

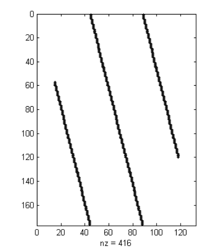

Fig 1. Sample structure of **D**.

RESULTS AND DISCUSSION. As expected, the effects of regularization are most obvious with decreased conditioning of the design matrix. Pixel-wise regularization tends to over-regularize and noise-reduction has to be paid off by increased residual aliasing. With reduced conditioning and stronger regularization the solution of methods that rely on regularization images can become dominated by the regularization image. Our new column-wise method takes advantage of the continuity of data and is virtually immune to such influences (fig 2). Such an artifact is clearly apparent in the result of method (iii) by the inclusion of the ellipse feature in the unfolded image (fig 2c) which only exists in the regularization image. In vivo images with a reduction factor of 6 using 8 coils show a clear SNR improvement made by the column-wise method (fig 3d) over the unregularized pixel-wise method (fig 3a). In addition, there is significant residual aliasing artifact reduction as compared to the regularized pixel-wise methods (figs 3b and 3c). Given the high reduction factor, the potential of this new approach is clearly apparent.

CONCLUSIONS. We have presented a line-by-line SENSE reconstruction technique that exploits the continuous behavior of the solution vector by incorporating derivative constraints into the regularization. Significant improvements in eliminating residual aliasing artifacts and noise enhancement are shown for phantom and in vivo data for severely ill-posed cases, i.e. when the reduction factor approaches the number of coils. Operating on a larger design matrix and using a more appropriate penalty term clearly outperforms previous methods. Therefore, this method shows promise in allowing for greater scan acceleration for a fixed receiver system.

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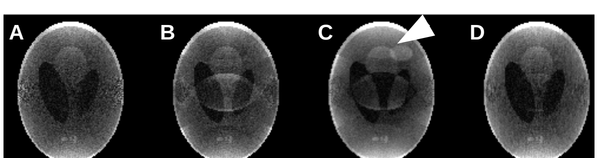

Fig 2. Phantom data (R=3, n_c=6) reconstructed using conventional SENSE (A), conventional SENSE with Tikhonov (B), conventional SENSE with Tikhonov and a regularization image (C), and column-wise SENSE with damped generalized SVD and generalized cross-validation (D). Tikhonov (B) reduces noise at the cost higher residual aliasing. In (C) the regularization image strongly affects the final solution (arrow). Regularized column-wise SENSE (D) gives the most optimal solution with regard to artifact power and noise amplification.

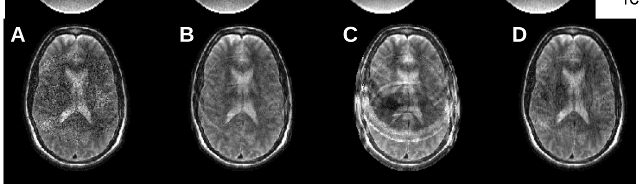

Fig 3. T2w-FSE data (R=6, n_c =8) reconstructed using conventional SENSE (A), conventional SENSE with Tikhonov (B), conventional SENSE with Tikhonov and a regularization image (C), and column-wise SENSE with damped generalized SVD and generalized cross-validation (D) (from left to right). The regularized column-wise SENSE clearly outperforms all other methods at these high reduction factors.