# Image Reconstruction in SNR Units

P. Kellman[1], E. R. McVeigh[1]

[1]NHLBI/NIH, Bethesda, MD, United States

## Introduction

The method for phased array image reconstruction of uniform noise images [1] may be used in conjunction with proper image scaling as a means of reconstructing images directly in SNR units. This facilitates accurate SNR measurement on a per pixel basis. This method is applicable to root-sum-of-squares magnitude combining, B1-weighted combining, and parallel imaging. A procedure for image reconstruction and scaling are presented, and the method for SNR measurement is validated with phantom data. Alternative methods that rely on noise only regions [2,3] are not appropriate for parallel imaging where the noise level is highly variable across the field-of-view. The proposed method is quite general and has a number of benefits.

## Methods

Roemer, et al [1] formulated equations for phased array combined image reconstruction for both root-sum-of squares (RSS) magnitude and optimum B1-weighted combining, and Pruessmann, et al. [4] formulated equations for parallel imaging using the image domain SENSE method. The equations [1] and [2] for SNR scaled images follow Roemer's formulation, where $SNR$ is the pixel intensity in SNR units, $\mathbf{p}$ is the vector of complex image values for each coil, $\mathbf{b}$ is the vector of complex coil sensitivities, and $\mathbf{R}_n$ is the noise correlation matrix which may be estimated from pre-scan acquisition of noise only data, $\mathbf{n}$, with components, $R_{ij} = (1/N)\sum_N n_i n_j^*$. It is further assumed that the scaling

$$SNR_{RSS} \approx \sqrt{\mathbf{p}^T \mathbf{R}_n^{-1} \mathbf{p}^*} \qquad [1]$$

$$SNR_{B1-weighted} = (\mathbf{p}^T \mathbf{R}_n^{-1} \mathbf{b})\big/\sqrt{\mathbf{b}^T \mathbf{R}_n^{-1} \mathbf{b}^*} \qquad [2]$$

$$SNR_{SENSE} = (\mathbf{u}^T \mathbf{p})\big/\sqrt{\mathbf{u}^T \mathbf{u}^*} \qquad [3]$$

of image values $\mathbf{p}$ and noise have the same effective scale factor in the image domain, as described below. The $SNR_{RSS}$ Eq. [1] is a very good appoximation at high SNR (>10) and may be further corrected to provide a good estimate at low SNR. In the case of SENSE [4], described by Eq.[3], $\mathbf{u}$ represents the vector of unmixing coefficients which are a reformatting of the unmixing matrix $\mathbf{U} = \left(\mathbf{S}^H \mathbf{R}_n^{-1} \mathbf{S}\right)^{-1} \mathbf{S}^H \mathbf{R}_n^{-1}$ which contains the optimum noise weighting ($\mathbf{S}$ is the coil sensitivity matrix). The unmixing vector is reformatted to be applied as a phased array combiner to the full FOV images reconstructed with zerofilling of undersampled data.

The pixel value, $SNR$, may be reconstructed in SNR units provided that care is taken in image and noise scaling. Each signal processing step may change the noise standard deviation. The overall noise scale may be computed or, alternatively, each step may be scaled to maintain a unity noise gain. For example, a standard FFT ($\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$) with input noise having standard deviation $\sigma$ will have standard deviation $\sigma\sqrt{N}$ after FFT, thus a scale factor of $1/\sqrt{N}$ must be applied for unity noise gain (note the value of N is the number of actual data samples and does not include zero-padding or zero-filling). Window functions, w(n), must likewise be scaled by their root-mean-squared value, $\sqrt{(1/N)\sum w^2(n)}$. The noise equivalent bandwidth of the combined analog and digital filter response of the receiver must also be measured and included into the noise scaling. This is easily measured from noise only data by measuring the mean squared value of average power spectrum normalized by the response at the center. Fig.1 shows the noise spectrum for 12288 averages for 2x readout oversampling. The noise equivalent receiver bandwidth factor is 0.79.

Validation of SNR scaled image reconstruction was performed using a time series of 256 phantom images acquired using a Siemens Sonata 1.5T scanner using a single-shot TurboFLASH sequence. Both full k-space imaging with RSS combined magnitude and parallel imaging using rate 2 SENSE were performed. Pre-scan noise and raw data were acquired and reconstruction was performed off-line using Matlab. SENSE g-factors [4] were estimated from the prescan noise and the B$_1$-maps and compared with the direct measurement of standard deviation images.
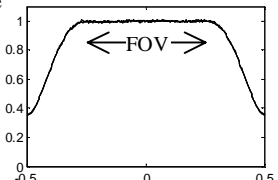


Fig 1. Noise spectrum

## Results

The SNR= mean/std dev was measured from N=256 images for each pixel. The (a) mean SNR scaled image, (b) std dev, and (c) SNR measurement are shown in Fig. 2 and 3 for RSS and SENSE combining, respectfully. The mean SNR scaled images of Fig.2,3 (a) agrees with the SNR measurements of Fig.2,3 (c) within 5% (note (a) and (c) are window-leveled the same). This example used only 4 coils to illustrate a higher g-factor. Fig. 3(d) shows a more standard reconstruction without scaling for uniform noise with corresponding noise map (e).

## Discussion

The proposed method is broadly applicable to a number of image reconstruction methods. It has a number of advantages over using a noise only region to estimate background noise [2,3]. The use of prescan noise avoids contamination of the noise region by signal artifact, may use a large number of pre-scan noise which reduces the fluctuation of the noise estimate, and properly accounts for the noise correlation between array elements.

## References

[1] Roemer PB, et al. MRM. 1990;16:192–225.
[2] Henkelman RM. Med Phys. 1985;12:232–233.
[3] Constantinides CD, et al. MRM. 1997;38:852–857.
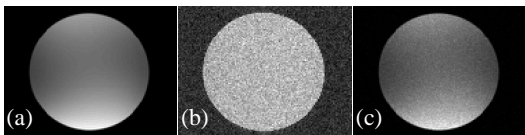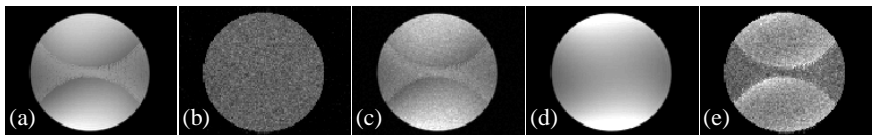[4] Pruessmann, et al. MRM. 1999; 42(5): 952-62.

Fig 2. RSS combined images

Fig. 3. SENSE combined images