# Noise Amplification in Non-Cartesian Parallel Imaging

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## Abstract

The noise amplification inherent in parallel imaging is investigated as to the influence of the k-space sampling pattern employed. Its spatial variation was simulated for various coil configurations, reduction factors, and acquisitions methods. This involved a separate estimation of the standard deviation for each pixel in a series of images reconstructed with an iterative algorithm from sets of noise samples. Compared to Cartesian acquisitions, radial and spiral ones are shown to yield a more uniform noise amplification and a smaller maximum loss in signal-to-noise ratio. Hence, they promise to provide superior image quality in parallel imaging, in particular with high acceleration.

#### **Introduction**

Parallel imaging is ultimately limited by signal-to-noise ratio (SNR) constraints [1,2]. The space variant noise amplification induced by image reconstruction consequently constitutes a crucial criterion, not only in the design of receive coils, but also in the selection of scan parameters. Using the sensitivity encoding (SENSE) approach [3], its analytical quantification for a given coil configuration and scan geometry is simple for Cartesian acquisitions, but involves impracticable computational effort for non-Cartesian acquisitions [4]. Mainly for this reason, the choice of the *k*-space sampling pattern has not been guided by the predicted noise amplification yet. Moreover, a consistent comparison between Cartesian and non-Cartesian parallel imaging has not been carried out in this respect to date.

## Methods

Instead of analytically calculating the noise amplification, we estimated its value based on noise images [5]. For this purpose, we placed a variable number of surface coils equidistantly along the circumference of a slightly enlarged circular field of view. We simulated their sensitivities, and synthesized noise data with Gaussian distribution, assuming no correlation between individual samples. We then reconstructed sets of noise data separately, using an iterative algorithm [4], and calculated the standard deviation for each pixel. The results were normalized to obtain a constant standard deviation equal to one for a full Cartesian acquisition, and their scaling was adjusted to the reduction factor. Thus, a direct comparison between different k-space sampling patterns was facilitated. The required number of iterations was determined with phantom data.

# **Results**

Representative maps of the noise amplification obtained for a configuration with 8 coils are shown in Fig. 1. For Cartesian acquisitions, the simulated values deviated from analytical ones by less than 2% on average, using 400 sets of noise data. Based on results like those given in Fig. 2, only a circular *k*-space shutter, but not a sampling density compensation were employed in the iterative reconstruction. The noise amplification then closely matched theoretical predictions for a direct reconstruction of full acquisitions [6]. A quantitative analysis of selected maps of Fig. 1 is provided in Fig. 3.

## **Conclusions**

The maps for the radial and spiral acquisitions still reflect the coil configuration, but, in general, exhibit less spatial variation of the noise amplification. The comparison of the histograms reveals that the maximum decrease in SNR

is substantially reduced despite the penalty for sampling *k*-space nonuniformly. How regularization affects these results remains to be investigated.

#### **References**

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Fig. 2. Average noise amplification for a full radial acquisition. The iterative reconstruction optionally included a circular k-space shutter (dashed, solid) or a sampling density compensation (dashed).



Fig. 1. Maps of the noise amplification for (from top to bottom) Cartesian, radial and spiral acquisitions, using (from left to right) reduction factors of 4.0, 4.9, and 5.8. The scaling was adjusted individually.



Fig. 3. Histogram of the noise amplification for Cartesian (solid) and spiral (dashed) acquisitions, using a reduction factor of 4.0 (top) and 5.8 (bottom).