Reduced Gibbs' Ringing with *k*-Space Extrapolation by Sensitivity Encoded Linear Prediction

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Introduction

k-space extrapolation by linear prediction (LP) is a well-known concept to reduce Gibbs' ringing and to allow a limited amount of superresolution [1]. LP extrapolation is not currently in wide use: the computational cost at the time of its emergence was comparatively high; also, this type of extrapolation can be unstable when too much improvement is attempted. Here we demonstrate how sensitivity encoding similar to parallel imaging can be incorporated to improve the LP extrapolation approach.

Both LP extrapolation and parallel imaging reconstruction – especially *k*-space based methods – generate "missing" *k*-space lines from a linear combination of measured ones. The two approaches differ in the relative arrangement of support and target points.

Linear prediction uses weights w_n, one set for each channel, for the estimation

$$(1) \qquad s(k_y) = \sum_{n=1}^{N} w_n \cdot s(k_y - n \cdot \Delta k)$$

to generate one missing sample from *N* measured (or previously estimated) samples. Figure 1 shows a typical *k*-space pattern for linear prediction with *N*=4 source points.

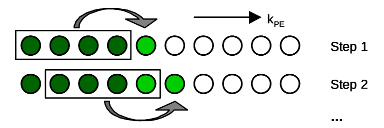

Figure 1: Linear prediction (LP) extrapolation. In a first step, *N* measured source samples ● (here *N*=4) are used to estimate one unknown sample ●. In subsequent steps, measured and estimated samples are used to successively predict the other unknown samples ○.

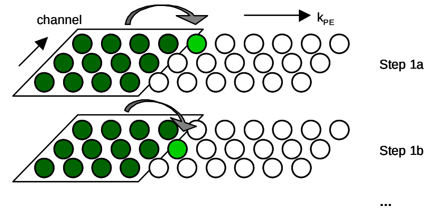

Figure 2: Sensitivity encoded LP. *N* source samples in each of the *C* channels (here *C*=3) are used to estimate one unknown sample in one channel. This is repeated for the missing samples in the other channels before moving on to step 2 ff. (cf. fig. 1).

Materials and Methods

Instead of using *N* source samples to estimate one target sample, sensitivity encoded (SE-) LP uses *NC* source samples $w_{n,c,cout}$ for each output channel c_{out}, where *C* is the number of receiver channels involved, to estimate one target sample in a single channel c_{out} according to

$$(2) \qquad s(k_y, c_{out}) = \sum_{c=1}^{C} \sum_{n=1}^{N} w_{n,c,cout} \cdot s(k_y - n \cdot \Delta k, c) \quad .$$

Thus the spatial encoding capability of receive coil arrays with spatially varying sensitivity patterns, as it is used in standard parallel imaging, is utilized to assist and stabilize the linear prediction process. Figure 2 shows the modified usage of data with *N*=4 and *C*=3 channels.

Simulations of a numerical Shepp-Logan phantom and 4 synthetic Gaussian-shaped sensitivity profiles were used to validate the method. The *k*-space matrix size was 128 x 128 with white noise of –21 dB added to investigate the stability of the method. Both standard LP extrapolation in each channel individually, and the proposed SE-LP extrapolation were performed with *N*=2 to fully regenerate all 128 lines.

In-vivo head measurements of a healthy volunteer with an axial T_2-weighted turbo-spin echo (TSE) sequence and a 12-channel head array (Head Matrix, Siemens Avanto 1.5T) were used to verify the method's performance in practice. Base resolution of these data was 256; 231 acquired *k*-space lines were reduced to 128 lines, which were then extrapolated using both LP and SE-LP with *N*=2 to the full 256 lines.

No high pass filtering of the data was used (see [1]). The weights were calculated from a linear fit of the inner k-space data similar to GRAPPA [2]. The lower and upper halves of *k*-space were treated separately, and independent processing, after readout FT, was performed for each column.

Results

Figure 3 contains the phantom results. It can be seen that some of the Gibbs' ringing caused by the data truncation has been removed, to a lesser extent in standard LP extrapolation, and to a greater extent in SE-LP extrapolation. The root-mean-squared error of the latter was always 22 ±5 % better than the former, relative to the full data set for successively added extrapolated *k*-space lines. In SE-LP minor additional ringing was introduced (see arrow).

Figure 3: Synthetic phantom results. Reduced phase resolution (L-R) with 64 of 128 lines (left); standard LP extrapolation with *N*=2 to 128 lines (middle); SE-LP extrapolation with *N*=2 to 128 lines (right).

Figure 4 shows the in-vivo results. Here also, some of the Gibbs' ringing has been removed to a lesser extent for LP, and to a greater extent for SE-LP (see boxes).

There were no stability problems using *N*=2; for *N*=3 or higher both methods became unstable. Although Gibbs' ringing was reduced, no significant resolution improvement was observed. The computational efficiency of SE-LP is comparable with other parallel imaging methods.

Discussion and Conclusion

SE-LP appears to be a robust means for routine reduction of Gibbs' ringing. There are similarities to the GRAPPA operator formalism [3]. The latter, however, focuses on the removal of aliasing / undersampling artifacts, or on an increase in resolution, while typically *N*=1.

References

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[2] M.A. Griswold, *et al.*, *MRM* **47**, 1202 (2002)
[3] M. Blaimer *et al.*, *Proc ISMRM* **12**, 2417 (2004)

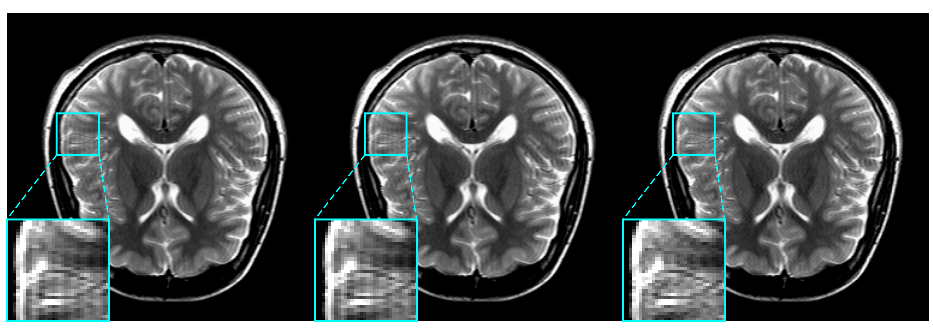

Figure 4: In-vivo results. Reduced phase resolution (L-R) with 128 of 256 lines (left); standard LP extrapolation with *N*=2 to 256 lines (middle); SE-LP extrapolation with *N*=2 to 256 lines (right).