Self-Calibrating Sensitivity Encoded Image Reconstruction using Rescaled Matrix Method for Non-Cartesian K-space Trajectory

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Introduction: Sensitivity encoding (SENSE) with non-Cartesian k-space trajectories was proposed, combining gridding with conjugate-gradient (CG) iteration (1). However, it is computationally expensive as well as prone to amplified noise to employ gridding operations with iterations. In this work, we propose several modifications of the CG approach using a rescaled matrix, speeding up the non-Cartesian SENSE reconstruction without gridding. In addition, the proposed scheme provides a tradeoff between image accuracy and noise during image progression with iterations.

Theory: Conventional SENSE reconstruction with non-Cartesian k-space is reduced to Eq. [1]: $(IE^H DEI)(I^{-1}v) = a,\ a = IE^H Dm$ [1]
where *I* is an intensity correction matrix, *D* a density correction matrix, *E* an encoding matrix, E^H its hermitian matrix, *m* a non-Cartesian k-space, and *v* a reconstructed image. The Eq. [1] is solved iteratively by CG method with forward and reverse gridding between Cartesian and non-Cartesian k-spaces.

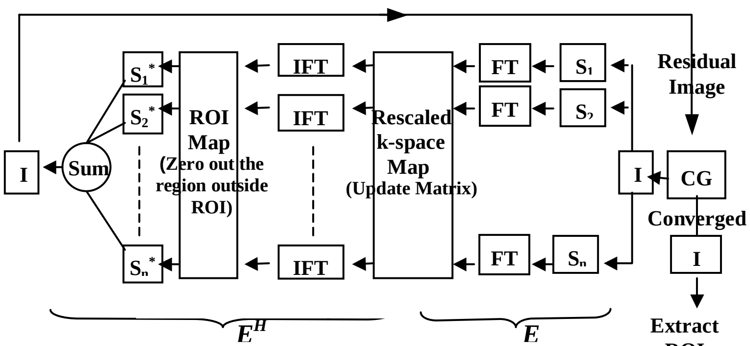

Fig. 1 A schematic of SENSE implementation with a rescaled matrix

To reduce computational complexity and gridding error propagation with iterations, we incorporate rescaled matrix method (2) into the CG loop as shown in a flow chart (Fig. 1): 1) A non-Cartesian k-space is expanded to *rN*x*rN* (*N*: readout matrix size) by a scale factor of *r* by rounding rescaled coordinate off to the nearest rectilinear grid location. A repeated mapping of several points onto the same coordinate is corrected by calculating their mean values. A rescaled k-space map in Fig.1 is reconstructed by setting the located coordinates to be ones and others to be zeros. 2) Coil sensitivity (S_n, *n*=coil index) is calculated as shown in (1) using low frequency signals and zero padding in the rescaled k-space. 3) In the E^H process, the rescaled k-space at each coil is inverse Fourier transformed (IFT), generating a reconstructed image in the central *N*x*N* region and severe aliasing artifacts outside of it. A region-of-interest (ROI) map is multiplied to replace the outer region by zeros. The ROI map is composed of ones in the central *N*x*N* image and zeros outside of it. 4) *D* matrix is removed over the entire process, because non-Cartesian k-space was already mapped onto a rescaled rectilinear grid in 1). After initialization with *a* in Eq. [1], a residual image (*rN*x*rN*) needs to be multiplied by $IE^H EI$. 5) In the *E* process, an intensity corrected residual image undergoes the multiplication of coil sensitivity followed by FT yielding rescaled k-space. The coil k-space is updated simply by multiplying the rescaled k-space map in 1). 6) Once the CG loop is converged, a residual image is intensity corrected, and then ROI is extracted.

Method: A right coronary artery data (matrix=256x256, FOV=250x250 mm^2, TR/TE/ flip angle =3.5ms/1.7ms/60^o, slice thickness=3 mm) was fully acquired using steady state free precession with a radial trajectory on 1.5T whole body MR scanner (MAGNETOM Sonata, Siemens Medical Solutions, Erlangen, Germany). Image progression during iterations in the rescaling SENSE (r = 4) was demonstrated as compared to conventional gridding reconstruction with a reduction factor (R) of 4. To measure the change of image accuracy, convergence ratio (δ) was calculated with the increase of iteration number (I_N) by Eq. [2]: $\delta = \sum \left|(IE^H EI)(I^{-1}v) - a\right| / \sum |a|$ [2]. Image reconstruction time for one iteration was measured for both the gridding SENSE and rescaling SENSE for comparison in the Matlab (MathWorks, Natick, MA, Pentium III 1GHz CPU).

Result: Conventional gridding results in severe artifact and noise that impede vessel depiction (Fig. 2a). As I_N is increased in the proposed rescaling SENSE implementation, images progress from low to high spatial resolution, but noise is also amplified (Figs. 2b-e). Convergence ratio, δ, rapidly decreases till $I_N \approx 10$, and is slowly reduced at $I_N > 10$. Compared with the image reconstructed by gridding (Fig. 2a), the SENSE image at I_N=9 (Fig. 2d) shows decreased residual artifact and noise. As R is increased, reconstruction time is not much changed for the rescaling SENSE, but rapidly decreased for the conventional gridding SENSE (Fig. 3).

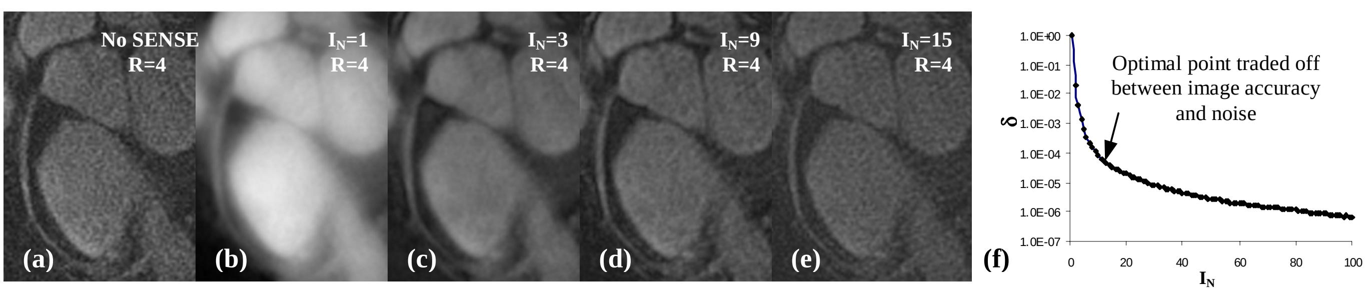

Fig.2. Image progression (b-e) in SENSE using rescaled matrix with the increase of I_N compared to gridding (a) with radial k-space, and δ versus I_N (f)

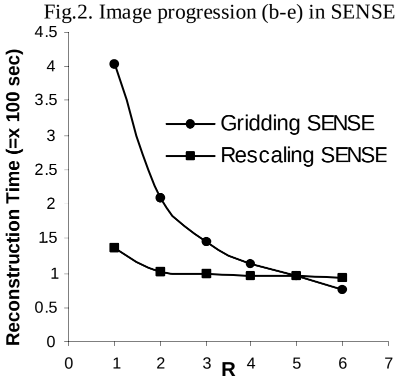

Fig.3. Comparison of reconstruction time between gridding SENSE and rescaling SENSE (r=4) for one iteration with the increase of R.

Discussion: The proposed SENSE implementation with a rescaled matrix has been successfully performed, speeding up image reconstruction and providing an automatic tradeoff between image accuracy and noise during iterations. Initializing *a* matrix in Eq. [1] without density compensation allows images to progress from low to high spatial resolution with the increase of I_N, but rounding-off errors in rescaling process makes noise gradually amplified. Optimal I_N can be determined by tracing the edge of L-shaped curve (Fig. 2f) to negotiate image accuracy with noise. I_N is, therefore, similar to regularization parameter in Cartesian SENSE (3). In conventional SENSE, gridding operation is the most computationally demanding, and becomes faster with the increase of R. However, reconstruction speed in the proposed scheme is highly dependent on the rescaling factor and FT. If the rescaling factor (r) is high, the rounding-off error is low, but FT has to deal with a larger size of matrix increasing the reconstruction time. Further investigation is needed to optimize the parameters, I_N and r.

References: 1. Pruessmann KP, et al. MRM 46: 638-651, 2001, 2. Oesterle C, et al. JMRI 10: 84-92, 1999, 3. Lin FH, et al. MRM 51: 559-567, 2004