Spatial modeling of the GRAPPA weights

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Introduction Compared to earlier k-space methods such as (VD)-AUTO-SMASH, GRAPPA[1,2] fits $N_{src}\times N_{coils}$ acquired "source" lines to each single coil ACS line. As for VD-AUTO-SMASH, tricks to make the weight estimation problem overdetermined are 1) "sliding" in ky and 2) grouping kx points (or rather "x"-points since Fourier transformation along kx is normally applied initially) together into blocks. Ultimately, a single set of weights could potentially be used across the entire x-FOV, yet, this is likely to be sub-optimal because the coil sensitivity normally changes significantly over the FOV. In this work, the artifact power for various block sizes have been investigated, for completeness with and without prior FFT in kx (denoted here as "FFTx"), and compared with a new proposed method for determining the GRAPPA-weights in x/ky space. In the latter, the key point is that each weight in the weight-set is a smooth parameterized function across *all* x values, rather than a scalar applicable to a given block of x-values.

Theory Let $\mathbf{y} = \mathbf{Aw}$, where $\mathbf{y}$ is a Nx×1 column vector containing the entire Fourier transformed ACS line. First assume we have independent weights for each x, then $\mathbf{w}$ is a $(N_{src}\times N_{coils}\times N_x)\times 1$ vector of weights to be reconstructed. Ignoring sliding in ky (in this derivation), the $\mathbf{A}$ matrix to be inverted is now sparse and heavily under-determined by a factor of $N_{src}\times N_{coils}$.

$$\mathbf{y} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{Nx} \end{bmatrix}}_{\text{coil j}} = \underbrace{\begin{bmatrix} \mathbf{a}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{a}_2 & 0 & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{a}_{Nx} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_{Nx} \end{bmatrix}}_{\mathbf{w}}, \text{ where } \begin{cases} \mathbf{a}_m = \left[\mathrm{S}_{\text{src1, coil1}}\ \mathrm{S}_{\text{src1, coil2}} \cdots \mathrm{S}_{\text{src1, Ncoils}}\ \mathrm{S}_{\text{src2, coil1}}\ \mathrm{S}_{\text{src2, coil2}} \cdots \mathrm{S}_{\text{Nsrc,Ncoils}}\right] \\ m = 1 \rightarrow Nx \end{cases} \quad [1]$$

and where $\mathbf{w}_m$ is a $(N_{src}\times N_{coils})\times 1$ vector corresponding to the mth x-position. Now, define the matrix $\mathbf{W}$ (size = $(N_{src}\times N_{coils})\times N_x$), where $\mathbf{w} = \mathrm{vec}(\mathbf{W})$ (NB: The vec-operator forms a matrix into a vector, column-by-column). For most scenarios, the coil sensitivity will vary smoothly along x, and so should the weights across the columns of $\mathbf{W}$. To enforce this, the coil weights are now modeled by a cosine basis set of order N_{order}, contained in a $N_x\times N_{order}$ matrix $\mathbf{C}$. We have

$$\mathbf{W} = (\mathbf{CH})^T \quad [2]$$

where $\mathbf{H}$ is the coefficient matrix for the cosine basis set, $\mathbf{C}$, forming $\mathbf{W}$. Instead of estimating the weights, vec($\mathbf{W}$), directly we want to estimate $\mathbf{H}$ containing a total of $N_{order}\times N_{src}\times N_{coils}$ unknowns. Combining Eqs. [1] and [2], gives

$$\mathbf{y} = \mathbf{A}\,\mathrm{vec}\left((\mathbf{CH})^T\right) = \mathbf{A}\,\mathrm{vec}\left(\mathbf{H}^T\mathbf{C}^T\right) = \left[\begin{matrix}\text{Algebra rule:} \\ \mathrm{vec}(\mathbf{XY}) = \left(\mathbf{Y}^T \otimes \mathbf{I}\right)\mathrm{vec}(\mathbf{X})\end{matrix}\right] = \mathbf{A}\underbrace{(\mathbf{C}\otimes\mathbf{I})}_{\mathbf{Q}}\underbrace{\mathrm{vec}\left(\mathbf{H}^T\right)}_{\mathbf{h}} = (\mathbf{AQ})\mathbf{h} \quad [3]$$

where, $\mathbf{Q}$ is the Kronecker product of $\mathbf{C}$ and the identity matrix $\mathbf{I}_{Nsrc\times Ncoils}$, and $\mathbf{h}$ is the vectorized representation of $\mathbf{H}^T$. Thanks to that both $\mathbf{A}$ and $\mathbf{Q}$ are very sparse, their product is easily calculated. Now we have arrived at the final least squares expression for the estimation of the cosine coefficients of the GRAPPA weights

$$\underbrace{\mathbf{h}}_{[(\text{Ncoils*Nsrc*Norder})\times 1]} = \underbrace{(\mathbf{AQ})^+}_{[(\text{Ncoils*Nsrc*Norder})\times \text{Nx}]} \underbrace{\mathbf{y}}_{[\text{Nx}\times 1]} \quad [4]$$

where "$^+$" denotes the pseudo-inverse. To reconstruct a missing ky line, the acquired nearby source lines, formed as $\mathbf{A}$ in Eq 1, is simply multiplied with $\mathbf{Qh}$.

Material & Methods A brain phantom was scanned with 256×256 resolution and 4 mm axial slices using an FSE sequence on a 1.5T GE EXCITE scanner. An 8-channel brain RF-coil was used. A single slice was first reconstructed using all k-space data. Then 75% of the ky lines was removed (i.e. ORF=4) outside of the center 20 lines used for estimating the weights. The weights were estimated using the standard block method (with and without Fourier transformation along kx) and with the method proposed in this work. Two "ACS blocks" were used for both approaches, forming the pattern [src-acs-acs-acs-src].

Results The artifact power, measured as the sum-of-squares difference between the GRAPPA reconstructed images and the fully sampled reference image, is shown in Figure 1. Without FFTx (dashed-dotted line), the fits fall far from the optimal solution due to the unresolved coil sensitivity in kx and the lowest artifact power is twice that of the block approach with FFTx. Using few blocks, the problem is highly overdetermined, yet producing more artifacts than the optimal 2^6=64 blocks (with FFTx). For the proposed basis set approach, 2^3 = 8-order set (see Fig. 1) seems to be sufficient to model the variation of the weights in x. Most interestingly, one obtains 20% less artifact power with the proposed method than with the optimal blocked one. In Figure 2, a cropped part of one of the coil-images is shown, with a) the fully sampled reference, b) 16-block (no FFTx), c) 16-block, d) 64-block, and e) 256-block GRAPPA. Finally, in f), a 16-order cosine GRAPPA is shown. b), d) and f) show images at optimum settings for the compared techniques, where the noise is markedly lowest in f).

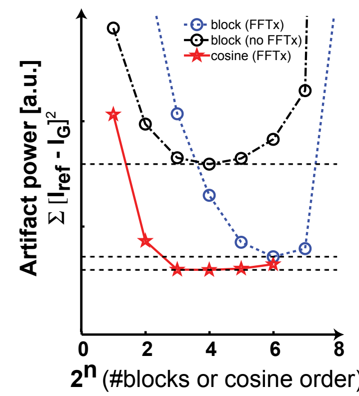

Figure 1. Artifact power vs. 1) block size (blue dashed line), 2) block size without initial FFT in kx (black dash-dotted line), and 3) cosine order (red solid line). Horizontal lines indicates the lowest artifact power. ORF=4

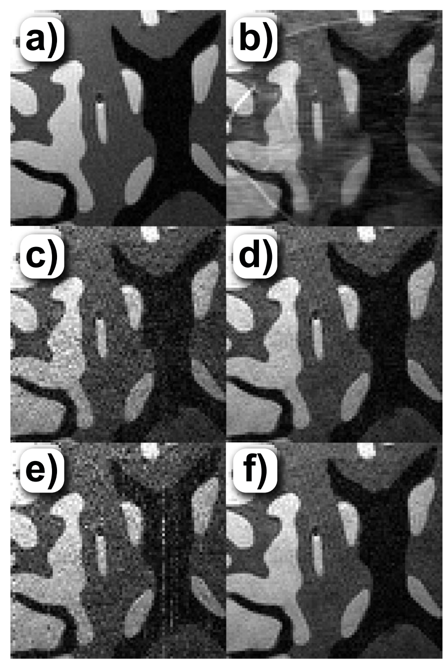

Figure 2. a) Reference b) 16 block (noFFTx) c) 16 block (FFTx) d) 64 block (FFTx) e) 256 block (FFTx) f) 16th order cosine. ORF=4

Discussion & Conclusion A new GRAPPA reconstruction algorithm has been developed, which has better artifact characteristics than the standard blocked approach, at least on the coil tested in this work. A low order cosine model works well, but this could easily be replaced in Eq. 3 by some other basis set depending on the coil configuration. Other coil configurations and k-space sampling strategies will be investigated.

References

1. Griswold et al., ISMRM Glasgow, #8 (2001)
2. Griswold M, MRM 47:1202-10 (2002)

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