Parallel MRI Reconstruction Using B-Spline Approximation

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Introduction

In parallel MRI, *k*-space is sampled less dense than in conventional spin warp MRI which allows reducing the total acquisition time by factors of 2 or more. In this work, a parallel MRI reconstruction algorithm using B-splines (PBS algorithm) is presented. The PBS algorithm is designed to minimize the reconstruction error.

State of the art

The reduced array-coil image is an original image modulated by the coil sensitivity and aliasing. The modulation by the coil sensitivity $S_l(x,y) = S(x,y)C_l(x,y)$ (Eq. 1) and aliasing $S_l^A(x,y) = \sum_{m=0}^{M-1} S_l(x, y + m\frac{N_y}{M})$ (Eq. 2) are both linear transformations, where S_l are the array-coil images, S_l^A are S_l with aliasing, S is the ideal reconstruction and M is the reduction factor. Assuming the invertibility of this composite transformation, the reconstruction consists of finding a proper inverse linear transformation $S(x, y + m\frac{N_y}{M}) = \sum_{l=1}^{L} \alpha_{lm}(x,y) S_l^A(x,y)$ (Eq. 3) with weights α_{lm}, where L is the number of array coils. The reconstruction can be performed in either image or *k*-space domain. The image domain method SENSE [1] estimates the sensitivities C_l with the knowledge of S and S_l (Eq. 1). An *LxM* sensitivity matrix is created for each pixel. The value at *(m,l)* in the sensitivity matrix is set to $C_l(x, y + m\frac{N_y}{M})$. Coefficients $\alpha_{lm}(x,y)$ are determined from the inversion of the appropriate sensitivity matrix.

The *k*-space methods (e.g. GRAPPA [2]) reconstruct the missing lines $s_j(k_x, k_y + m\Delta k_y)$ as a linear combination of several acquired lines in all coils $s_j(k_x, k_y + m\Delta k_y) = \sum_{l,n} \alpha_{jn,lm}(k_x) s_l(k_x, k_y + n\Delta k_y)$ (Eq. 4), where s is the *k*-space image. The coefficients $\alpha_{jn,lm}$ are estimated in the least square sense using the Eq. 4, where for several k_y was the line $s(k_x, k_y + m\Delta k_y)$ acquired as an auto-calibration line.

PBS Algorithm

The PBS method operates in the image domain. The reconstruction coefficients are determined directly without estimating the coil sensitivity maps. Taking into account that value in an aliased image S_l^A is a combination of M pixel values of the original images (Eq. 2) and the fact that the intensity values in two distinct pixels are independent, we define the ortogonality condition for coefficients α_{lm} (Eq. 3)

$$\sum_{l=1}^{L} \alpha_{lm}(x,y) S_l(x, y + m'\tfrac{N_y}{M}) = \begin{cases} S(x, y + m\frac{N_y}{M}) & for \quad m = m' \\ 0 & for \quad m \neq m' \end{cases} \qquad \text{(Eq. 5)}$$

The sensitivities and, thus, the reconstruction coefficients are smooth in space. It is not required to estimate the coefficients independently in each pixel as it is usually done. Instead, we look for the solution in a restricted space represented by a B-spline basis. This has a regularization effect and is advantageous from the computational point of view as well. The coefficients α_{lm} are represented as $\alpha_{lm}(x,y) = \sum_{i,j} g_{mijl} \beta^p(\frac{y}{h_y} - i)\beta^p(\frac{x}{h_x} - j)$ (Eq. 6), where g_{mijl} are B-spline coefficients and β^p are B-splines functions of order p with knot spacing h_y and h_x. The B-spline coefficients are estimated in order to minimize the reconstruction error Eq. 3.

The reference images used in Eq. 5 can be retrieved in full resolution in a pre-scan or in low resolution during the accelerated scan. Additional lines are acquired near the *k*-space center to form a low resolution image, while the outer part of the *k*-space is sampled more sparsely. The method is also capable of estimating the original image S_l for each coil separately. This is done by replacing the reference image S by S_l in (Eq. 5). The individual array-coil images are combined using the conventional sum-of-squares reconstruction, which provides an optimal SNR, when the coil sensitivities are unknown [5].

Measurements

The method was tested on phantom images acquired by a Siemens Magnetom Symphony 1.5T scanner using an 8 channel head coil. A FLASH pulse sequence was used with the following parameters: TR = 30 ms, TE = 8.7 ms, ϑ = 25°, FOV = 150x150 mm^2, slice thickness = 5 mm and matrix = 512x512. Reconstruction was done with an acceleration factors 2 and 4 with 24 extra reference lines. Images were reconstructed with the PBS algorithm an compared to the manufacturer-provided methods GRAPPA [2] and mSENSE.

Results and Discussion

In Fig. 1 the PBS algorithm is compared with mSENSE and GRAPPA. The mSENSE image showed the lowest SNR, however without significant artifacts. The SNR was higher in the GRAPPA image, however, there were many visible artifacts. Images reconstructed by the PBS method (Fig.1c) showed both no visible artifacts and a high SNR. The excellent reconstruction properties of the PBS algorithm (cf. Fig. 1) are also seen in the reconstruction error $E = \sum_{x,y} \left\| S(x,y) - \hat{S}(x,y) \right\|^2$, where $\hat{S}$ is the reconstruction and S is the sum of squares reconstruction of a fully sampled dataset.

The proposed PBS algorithm uses the B-spline regularization, which significantly reduces the number of variables as compared to SENSE. Furthermore, the reconstruction is not computed independently in each point as in SENSE. The reconstructed images have lower reconstruction error than two commercially used methods mSENSE and GRAPPA.

The method imposes no limitations on the coil configuration. It is robust to noise during the estimation. The future development will include a statistical approach considering the noise properties during the estimation and the reconstruction.

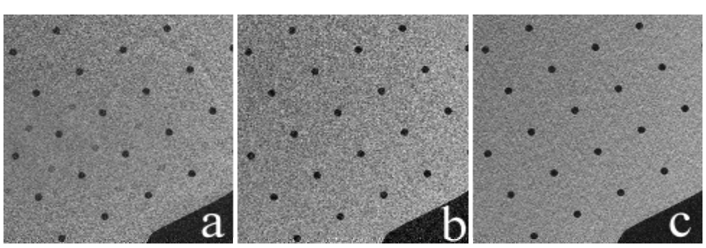

Fig. 1. Reconstructed images with error E for an acceleration factor 2. E4 is error for an acceleration factor 4. a) GRAPPA, E=170, E4=342; b) mSense, E=278, E4=1228;c) PBS, E=101, E4=771.

References

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