Efficient Variable Density SENSE Reconstruction

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Introduction

Variable density k-space sampling is sometimes used in parallel imaging to reduce aliased energy at low spatial frequencies (1) or to allow self-calibration using Nyquist-sampled data around the center of k-space (2). Variable density data can be reconstructed using SPACE RIP (1), Generalized SMASH (3), Generalized SENSE (4), GRAPPA (5) or by unwrapping only a uniformly undersampled subset, transforming back to k-space, and combining it with the Nyquist-sampled data (6). A method developed by Madore (7) for SENSE with self-calibration places the measured data at the correct sampling intervals in k-space and then fills in missing data with zeros. The data are multiplied by the appropriate density compensation factors for each k-space region. Fourier transformation is then followed by Cartesian SENSE unwrapping (8) assuming a reduction factor appropriate for the undersampled (outer) part of k-space. Madore's method has the advantage of being very computationally efficient but can result in ringing artifacts due to the abrupt transition in k-space density.

Methods

The image quality of Madore's method can be improved with almost no reconstruction time penalty by a very simple extension. Consider a 1D example where the outer part of k-space has threefold undersampling and the center of k-space is Nyquist-sampled (Fig. 1, top). By appropriately choosing the sampling pattern, we can split the data into two interleaving sets, a high resolution set *RH* and a low resolution set *RL* (Fig. 1, center and bottom). Because of the uniform spacing of both the samples in *RH* and the "holes" in *RL*, both data sets can be reconstructed using uniform-density Cartesian SENSE. If *RH* and *RL* are multiplied by appropriate k-space windows *H* and *L* that compensate for the sampling density, the two unwrapped images can be added. The final reconstructed image *I* is

$$I = S\{FT[H \times RH + L \times RL]\} \quad [1]$$

where *FT*[ ] and *S*{ } represent 1D Fourier transform and 1D SENSE unwrapping operators, respectively. For example consider the k-space window

$$L = \begin{cases} 1 & \text{in the Nyquist sampled region} \\ 0 & \text{otherwise} \end{cases} \quad [2]$$

with

$$H = 3 - 2 \times L \quad [3]$$

For this choice of *L* the reconstruction is equivalent to Madore's method. However we can smooth the abrupt transitions in *H* and *L* to reduce the ringing. Consider

$$L = \left[1 + e^{(ky - c)/w}\right]^{-1} - \left[1 + e^{(ky + c)/w}\right]^{-1} \quad [4]$$

with *H* given by Eq. [3] and where *c* and *w* represent the radius of the Nyquist-sampled region and the transition width respectively. *L* and *H* are shown in Fig. 2 for $c = 32$, $w = 8$. The transition width *w* can be chosen to trade ringing for aliasing: increasing *w* decreases ringing but increases low frequency aliasing. Note that if a sharp transition is used ($w = 0$), then Eq. [4] becomes equivalent to Eq. [2]. It can be shown that this method requires the approximation that the SENSE unwrapping operator commutes with the convolution operator in image space. This approximation is also used in uniform-density SENSE reconstructions when k-space windowing is used to reduce Gibbs ringing or when low spatial resolution sensitivity data are used (9).

Results

Scan data were acquired on a commercial 1.5T scanner (GE Healthcare, Milwaukee, WI) using an eight-channel head coil (MRI Devices, Gainseville, FL). Fully sampled data (256x256 matrix) were decimated to give three-fold undersampling in the outer part of k-space with 64 Nyquist-sampled center lines (128 acquired *ky* lines, net reduction factor two). The Nyquist-sampled k-space lines were used to generate a low resolution coil sensitivity calibration. The data were then reconstructed using Eqs. [1], [3] and [4]. Figure 3 shows the results for two transition widths ($w = 0$, $w = 8$), demonstrating the reduction in ringing as the transition width increases. This method can be extended to reconstruct partial Fourier data by appropriately modifying *H* to overweight data conjugate to the additional missing *ky* lines.

Conclusions

Variable density Cartesian SENSE data can be efficiently reconstructed by extending the method of Madore to reduce ringing artifacts introduced by the sharp k-space density changes. k-Space sampling is modified to allow the data to be segregated into interleaving low and high frequency subsets. Each subset is multiplied by a density compensation window that also smoothes the sampling density transitions. The data are added and reconstructed using the conventional uniform-density SENSE algorithm. The reconstruction time is almost the same as for uniformly-sampled data.

References

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Fig. 1. Top: 28 samples of a variable density *ky* data set with threefold undersampling and eight Nyquist-sampled center points. Center: high resolution component *RH*, Bottom: low resolution component *RL*. X and 0 represent measured and unmeasured points respectively.

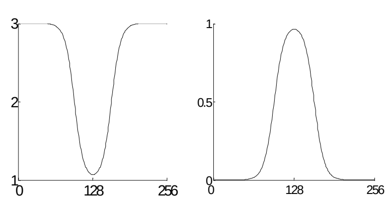

Fig. 2. *H* (left) and *L* (right) computed using Eqs. [3] and [4] for 256 *ky* samples with $c = 32$, $w = 8$.

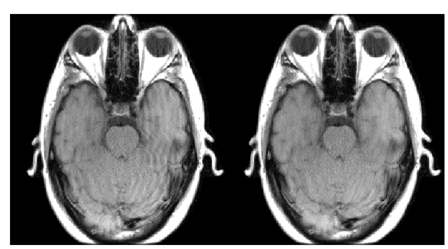

Fig. 3. Variable density scan with threefold undersampling and 64 center lines ($c = 32$) reconstructed with $w = 0$ (left), and $w = 8$ (right).