Disentangling the Effects of Varying Radial and Tangential Sampling Densities in Variable Density Spiral Imaging

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Introduction. High temporal resolution is crucial in MR imaging of rapid physiological phenomena. Spiral k-space sampling is therefore often used. Moreover, as MR images have most of the energy concentrated at the center of k-space, oversampling of this area may be advantageous. With variable density spiral k-space sampling [1], this may be achieved without penalty in temporal resolution by concomitant undersampling the high spatial frequencies in the radial direction. The undersampling of these high spatial frequencies in combination with the oversampling of the low spatial frequencies in the radial direction is known to induce limited aliasing artifacts [2]. While the variable sampling density in the radial direction has been addressed [2], the effects of varying the sampling density along the trajectory (i.e. tangentially) have, to our knowledge, not been fully described [3]. Here, we evaluate the effects of variable sampling density in both the radial and tangential directions using numerical simulations. In variable density spiral imaging, these two effects are necessarily combined. In order to disentangle both effects, pseudo Cartesian and circular sampling patterns are considered.

Material and methods. A Matlab algorithm was used to produce the k-space samples along the desired trajectories of the modified Shepp-Logan phantom. Image reconstruction was performed following a conventional gridding method (matrix size = 128*128) using a Kaiser Bessel convolution kernel (kernel width = 3, grid oversampling factor = 2). We define the Nyquist spatial frequency step (NSF) as the inverse of the desired field of view.
First, two images were produced from pseudo Cartesian sampling patterns: one undersampled in the horizontal (k_x) direction ($\delta k_x = 3/2 * NSF$) and one with a variable sampling density along that same direction ($1/2 * NSF \leq \delta k_x \leq 3/2 * NSF$) (Figure 1, a-b). The sampling step in the vertical direction was $\delta k_y = NSF$. Then, images were produced from circular sampling patterns. The radial sampling step was $\delta k_r = NSF$. The angular sampling step was chosen so that the tangential sampling step along the outer circle was $\delta k_{tang} = 2 * NSF$, $\delta k_{tang} = NSF$, and $\delta k_{tang} = 1/4 * NSF$ respectively (Figure 2, a-c).

Results. Figure 1 shows the Cartesian sampling case. Fig. 1c displays the classical aliasing artifact due to horizontal undersampling in k_x direction. If the central k-space region is properly sampled, the aliasing artifact is dramatically reduced (Fig. 1d, at the expense of increased noise. This can be seen on the profile taken along the horizontal direction (arrows on Fig. 1e-f).
Figure 2 shows the circular sampling case. As the angular sampling frequency increases, a high frequency artifact becomes more and more pronounced (Fig. 2d-f). The profiles (Fig. 2g-i) show a corresponding reduction in signal to noise ratio (e.g. in the highest signal intensity ellipse, it decreases from 65 to 7). This artifact does not vary significantly if any of the kernel width or the grid oversampling factor is doubled during gridding.

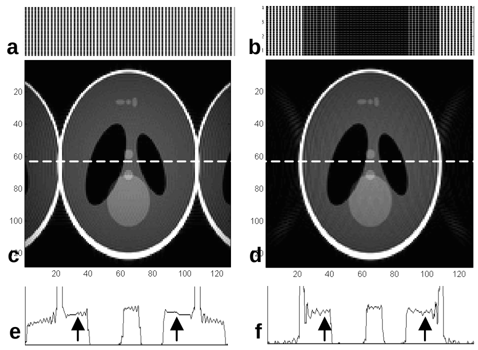

Figure 1. Comparison of two pseudo Cartesian patterns: (**a**) undersampled in the horizontal direction ($\delta k_x = 3/2 * NSF$) with (**b**) a variable-density ($1/2 * NSF \leq \delta k_x \leq 3/2 * NSF$) in that same direction. (**c**) and (**d**) are the images of the Shepp-Logan phantom obtained with these two trajectories. (**e**) and (**f**) show profiles taken along the white dotted line on (c) and (d).

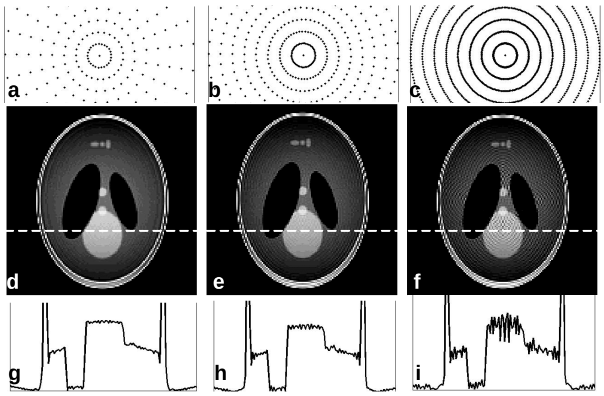

Figure 2. Images of the Shepp-Logan phantom ((**d**), (**e**) and (**f**)), with a circular field of view, obtained with circular sampling patterns. The tangential sampling density is (**a**) $\delta k_{tang} = 2 * NSF$, (**b**) $\delta k_{tang} = NSF$ and (**c**) $\delta k_{tang} = 1/4 * NSF$. Graphs **g-h** show profiles taken along the white dotted line on (d-f).

Conclusion and perspectives. Undersampling the high spatial frequencies in a variable sampling density trajectory can be used in the radial direction to reduce acquisition time without introducing considerable aliasing artifacts. In circular sampling patterns, tangential oversampling yields a strong calculus noise which seems to be independent of the gridding parameters. Moreover, tangential undersampling does not degrade the image quality and slightly increases the signal to noise ratio. This counter-intuitive effect might be due to the fact that the gridding kernel is circularly symmetrical while the k-space sampling is not. This asymmetry increases with the difference between the tangential and radial sampling densities.
In order to further clarify the origin of the artifact, we will use a reconstruction method without convolution kernel [4]. Simulations will further be completed by introducing thermal noise and k-space filtering due to transverse relaxation.

References

[1] Kim, D. H., Adalsteinsson, E., Spielman, D. M. (2003) *MRM*. 50, 214-219, **[2]** Tsai, C. M., Nishimura, D. G. (2000) *MRM*. 43, 452-458. **[3]** Pipe, J. G. (2000) *MRM*. 43, 867-75. **[4]** Rosenfeld, D. (1998) *MRM*. 40, 14–23.