Visualizing Steady-State Free Precession

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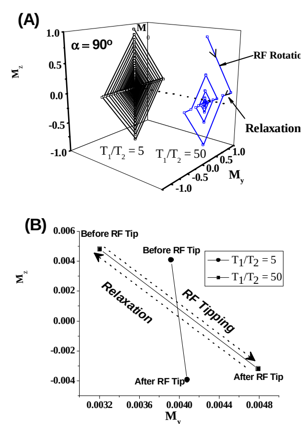
Introduction Improvements in hardware have now made steady-state free precession (SSFP) practical in clinical and research studies [1,2]. In addition to rapid acquisition of MR images with high signal-to-noise ratio, such advances have also allowed for fast measurements of relaxation constants [3,4] and *in vivo* maps of temperature [5] and blood oxygen saturation [6]. As SSFP methods are further refined to provide real-time physiological information, it is important to reflect on how such signals are established. While a sound mathematical understanding of SSFP exists in the literature, there has been much less effort to explore an intuitive understanding of how such signals are formed. In particular, a description of the interplay of the various imaging parameters determining the final steady-state signals is needed. There have been efforts to apply the elegant phase-graph method via the echo formation pathways [7]. The primary purpose of this work is to put forth a more intuitive pedagogical model, from the viewpoint of classical dynamics, for understanding how SSFP signals are formed. This work presents a geometric description in which the magnetization evolution towards steady state is developed. In so doing, we develop a model for understanding the interplay between relaxation constants, flip angle, time between RF pulses (T_R), off resonance ($\Delta\omega$), and phase-cycling patterns on the steady-state signal from a spin ensemble. In the simplest terms, SSFP signals result from repeated application of RF pulses that tip the total magnetization at a pre-determined flip angle and time interval. After the application of every RF pulse, the magnetization evolves by recovering along the longitudinal direction and losing coherence in the transverse plane. The natural tendency of the spins' drive towards thermal equilibrium encountering a repeated perturbation by the RF pulses leads to the establishment of a steady-state MR signal. In this state, there is a dynamic equilibrium between the natural relaxation process and the magnetic perturbation which leads to a free precession of the magnetization in the rotating frame.

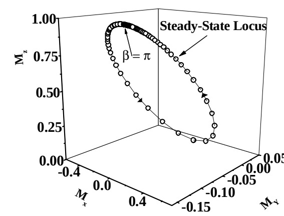
Steady-State Magnetization in the Absence of Off Resonance Let us consider the effect of repetitive application of RF pulses that tip the ensemble magnetization about the x axis by α in the presence of spin relaxation. In the first pulse, the initial magnetization (M_0) is rotated into a new position. Due to the nuclear relaxation process, the M_z recovers longitudinally and the transverse component, M_y, is reduced before the next RF pulse is applied. In this set up, $M_x = 0$ and it remains zero provided the phase rotation, defined as $\beta = \Delta\omega T_R$, is zero. Clearly, the extent of the longitudinal recovery and transverse loss are determined by the appropriate relaxation constants. Since $T_2 < T_1$, it is clear that the transverse loss will dominate; thus the tendency of the system is to reach a steady-state magnetization of zero. However, in practice T_2 and T_1 are finite, which means that there is always a non-zero magnetization that remains which is then tipped by the subsequent RF pulse. After a number of pulses, the RF tipping takes the magnetization from a point $\mathbf{r_0}$ in magnetization phase-space to another point $\mathbf{r_1}$ and the subsequent relaxation during T_R takes it back to $\mathbf{r_0}$. This condition is referred to as the steady state. Note that there are indeed two fixed points in phase space, one prior to the pulse and one after the pulse. This behaviour is shown for $\alpha = 90^o$ for two different T_1/T_2 ratios in Fig 1A. As shown in Fig. 1B, the greater the T_1/T_2 ratio, the greater the distance between the two fixed points in magnetization phase space.

Fig. 1



Steady-State Magnetization in the Presence of Off Resonance When β is non-zero, an intriguing event takes place. As β is increased, the trajectory in magnetization phase space moves out of the YZ plane. This is expected since the phase rotation effectively forces a non-zero M_x. This leads to the establishment of two coplanar fixed points with the plane determined by β. Under these conditions, to establish steady state, the effect of RF tipping needs to be negated through relaxation *and* phase rotation over a single T_R. In the literature, it is customary to take the magnetization immediately following the RF as the steady-state value of the magnetization. If we adopt this convention here and follow this point in phase space for different values of β over a full period of 2π, a cyclic locus of steady-state values can be identified. This is shown in Fig. 2 for $\alpha = 10^o$. One can relate the geometric properties of this locus to MR parameters: (1) The eccentricity of the cycle is related to T_1/T_2 - a circle when $T_1/T_2 = 1$ and an ellipse when $T_1/T_2 > 1$ (Fig. 3); (2) The plane of the locus is determined by α; (3) The points at equal intervals of β are not uniformly distributed on the locus. In particular, for any given α, the points are distributed densely around $\beta = \pi$ with the exact distribution determined by the T_1/T_2 and α; (4) As the MR signal is the orthogonal projection of the magnetization locus, it is expected that the MR signal will be dependent on the eccentricity and the plane of locus.

Fig. 2



Geometry of the SSFP Signal on the Transverse Plane The transverse magnetization can be related to the geometrical properties of the orthogonal projection of the steady state magnetization locus on the transverse plane. When $\beta = 0$, the orthogonal projection of the steady state magnetization yields a point in the transverse plane. From the perspective of classical dynamics, this state can be characterized as an unstable equilibrium since *any* static field inhomogeneity leads to an abrupt geometrical change that takes the point to a locus in the transverse plane. The length of the major and minor axes and the distribution of points over the locus are determined by T_1/T_2 and α. In general, for small $T_1/T_2 \mapsto 1$ and small α, the locus is an ellipse with its major axis along the x axis. As T_1/T_2 or α increase, the eccentricity of the ellipse decreases and the locus becomes a circle as $\alpha\ \zeta\ \pi$ or $T_1/T_2\ \zeta\ 4$. There are number of important physical implications that may be explained based on the properties of the transverse magnetization locus. It can be shown that the eccentricity of the ellipse determines the points at which the maximum and minimum signal values occur and also strongly influences signal homogeneity in presence of off resonance. For instance, when the eccentricity is nearly one, the signal is homogenous but the signal magnitudes are nearly zero. However, when the eccentricity is nearly zero, the signal is inhomogeneous for any range of off resonance. We showed that the optimum SSFP signal results when the eccentricity is $2^{-1/2}$. This implies that an optimum flip angle for an imaging application will be determined by the T_1/T_2 of the medium. One can identify the appropriate α for a given T_1/T_2 from the contour line with eccentricity ~ 0.7 shown in Fig. 4. This analysis also leads to the results that, as T_1/T_2 increases, the size of the transverse magnetization locus decreases, which leads to a reduction in peak signal value. Based on these geometric descriptions, it is also possible to provide an intuitive explanation for the characteristic features of the typical off-resonance frequency response of a spin ensemble (8).

Fig. 3

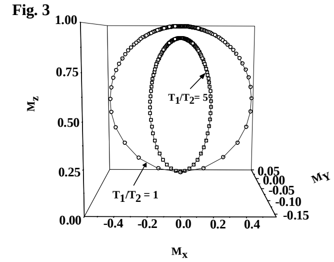
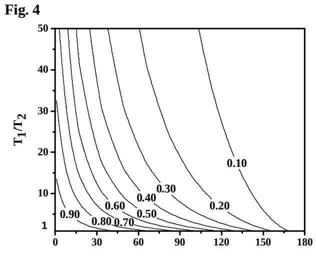


Fig. 4



Summary & Conclusion In this paper we demonstrate an alternate method for visualizing how SSFP signals are formed and how such signals are strongly influenced by the choice of MR parameters. In particular, we have shown that, when repeated RF pulses are applied to a system that undergoes relaxation during the repetition interval, a steady state is reached and this steady state may be thought of as an unstable equilibrium with respect to off-resonance frequency. When this equilibrium is disturbed, a steady state locus is formed in the three dimensional magnetization phase space. We have also shown that the projection of this locus on the transverse plane represents another locus that contains the sets of all points in the off resonance signal profile. Our analysis also showed that the major and minor axes of this locus are determined by α and T_R. Most importantly, we show that it is possible to understand SSFP signal properties based on the eccentricity and the size of the transverse magnetization locus.

References [1] Carr H. Phys Rev,1958;112:1693-1701. **[2]** Oppelt A et al. Electromedica,1986;54(1):15-18. **[3]** Scheffler K. MRM 2001;45:720-723. **[4]** Deoni SC et al. MRM, 2004; 52:435-439. **[5]** Scheffler K. MRM,2004; 51:1205-1211. **[6]** Dharmakumar R. MRM in Press **[7]** Scheffler K. Conc in MR,1999;11(5):291-304 **[8]** Haacke EM et al. *MRI: Physical Principles and Sequence Design.*1999.451-512.