A simple geometrical description of the TrueFISP transient and steady state signal

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Introduction: In this work, a simple pictorial approach is presented for describing the TrueFISP transient phase and steady state signal. It is based on simple geometrical considerations without the need for abstract mathematical treatment or further approximations. Short formulations are derived determining the direction of the magnetization vector for which a smooth monoexponential decay is obtained even at considerable off-resonance frequencies. Analytic expressions are developed which describe the signal evolution, accounting for T_1, T_2, flip angle and frequency offset.

Theory: If relaxation is neglected, successive RF pulses with alternating phase and flip angle α drive the magnetization vector $\boldsymbol{M}$ towards a state with its precession phase evolving approximately from $-\phi/2$ to $+\phi/2$ between the RF pulses. It follows from the depiction in Fig. 1 that the resulting ideal zenith angle θ between $\boldsymbol{M}$ and the z axis fulfills the condition

$$\tan\theta = \frac{\tan(\alpha/2)}{\cos(\phi/2)} \quad . \qquad [1]$$

If relaxation is included, the transverse component $M_\perp$ decreases between RF pulses, and the projection of the dephasing trajectory into the x-y-plane appears slightly asymmetric. According to Fig. 2, an MR signal with its echo center acquired at $TE = TR/2$ exhibits a small phase $d\phi$ given by

$$\tan d\phi = \frac{1-E_2}{1+E_2}\tan(\phi/2) \quad , \qquad [2]$$

with $E_{1,2}=\exp(-TR/T_{1,2})$. Further analysis is simplified if the evolution of $\boldsymbol{M}$ is assessed in a 2D coordinate system defined by the z axis and the transverse component of the magnetization vector. It can be shown that, in this plane, an α pulse has the approximate effect of mirroring $\boldsymbol{M}$ across the line defined by the ideal zenith angle θ. Once $\boldsymbol{M}$ is brought close to the θ line, its evolution is kept balanced close to this direction by successive relaxation intervals and RF excitations, yielding a smooth signal time course without fluctuations (Fig. 3). Evidently, Eq. 1 and Eq. 2 in good approximation determine the optimal initial direction for which signal fluctuations are avoided and thus represent the target of various preparation schemes described in literature (1-4).

In the steady state, the "gain" due to longitudinal relaxation must counterbalance the "loss" caused by transverse relaxation so that the length of $\boldsymbol{M}$ does not change over an TR cycle. This is the case if $\boldsymbol{\Delta M}_{relax}$ is perpendicular to $\boldsymbol{M}$. With this simple approach, the following analytic expression is obtained for $M_{SS,\perp}$ which may serve as an approximation for the MR signal:

$$M_{SS,\perp} = \frac{M_0(1-E_1)\tan(\alpha/2)\cos(\phi/2)}{(1-E_1)\cos^2(\phi/2)+(1-E_2)\tan^2(\alpha/2)} \quad . \qquad [3]$$

For assessment of the transient state, it is useful to analyze the vector difference between the actual and the steady state magnetization. If a smooth exponential signal decay is assumed, the magnetization $\boldsymbol{M}''$ after a complete TR cycle follows

$$\boldsymbol{M}'' - \boldsymbol{M}_{SS} = \lambda(\boldsymbol{M} - \boldsymbol{M}_{SS}) \quad . \qquad [4]$$

This equation system can directly be solved for the apparent relaxation rate λ. Besides the trivial solution $\lambda=1$ (for $\boldsymbol{M}=\boldsymbol{M}_{SS}$), the following result is obtained:

$$\lambda = \frac{\cos^2(\phi/2)\cdot E_1\cos^2(\alpha/2)+E_2\sin^2(\alpha/2)}{\cos^2(\phi/2)\cdot\cos^2(\alpha/2)+\sin^2(\alpha/2)} \quad . \qquad [5]$$

For $\phi=0$, this result reduces to an expression proposed for on-resonant magnetization (5). It is in agreement with formulations published recently, derived with a mathematical treatment based on perturbation theory (6). Combining these findings, using $\lambda = \exp(-T_1{}^*/TR)$ and $M_0=1$, the ideal signal time course can be described by an exponential decay:

$$S(t) = M_{SS,\perp} + (\sin\theta - M_{SS,\perp})\exp(-t/T_1^*) \quad . \qquad [6]$$

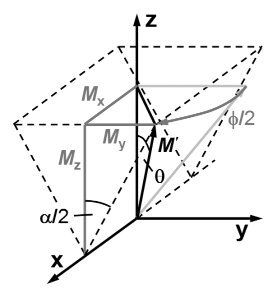

Fig. 1: Off-resonance dephasing and ideal zenith angle

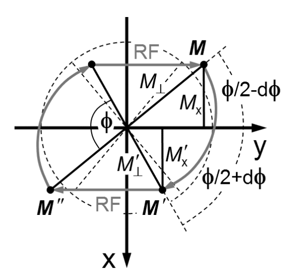

Fig. 2: Transverse relaxation and the TrueFISP signal phase

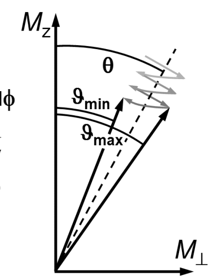

Fig. 3: Smooth evolution along the θ cone

Methods: For verification of the theory, both numerical simulations based on the Bloch equations and MR experiments were carried out. In the latter, transient signal time courses were sampled from a cylindrical phantom with T_1=1035ms and T_2=92ms. Different preparation pulses and various flip angles were tested. With an additional gradient of G_z = 0.04 mT/m in z direction, a position-dependent frequency offset varying linearly with the z coordinate was created. Data were acquired as 1D projections with the read direction oriented along the z axis so that a large range of off-resonances could be assessed with each individual experiment.

Results: Both simulations and experiments gave equivalent and consistent results. As illustrated in Figs. 4 and 5, simulation results (dots) are described by the closed form expressions of Eqs. 3 and 5 (solid) with high accuracy. The same is true for experimental data, as shown in Fig. 6. For considerably off-resonant magnetization (ϕ=19π/20!), the α/2 preparation scheme is suboptimal in that signal fluctuations are observed (solid gray). After preparation towards the respective ideal zenith angle, an essentially smooth signal decay is obtained (solid black) which is well reflected by Eq.6 (dotted black).

Discussion: A simple picture has been presented for assessment of the TrueFISP signal in the transient phase and in the steady state. Without the need for advanced mathematical treatment, analytical expressions were derived for the ideal initial direction of the magnetization vector, the steady state signal and the apparent relaxation time which determine the TrueFISP signal evolution. The results are potentially useful for contrast calculations and for off-resonance error corrections in TrueFISP-based relaxometry studies (7).

References: (1) Hargreaves et al. Magn Reson Med 46: 149-58 (2001). (2) Nishimura et al. Proc ISMRM 2000; #301. (3) Le Roux. J Magn Reson 163: 23-37 (2003). (4) Paul et al. Proc ISMRM 2004; #2663. (5) Scheffler. Mag Reson Med 49:781-783 (2003). (6) Ganter. Magn Reson Med 52: 368-375 (2004). (7) Schmitt et al. Magn Reson Med 51:661-667 (2004).

Fig. 4: Steady state signal $M_{SS,\perp}$ (theory and simulation)

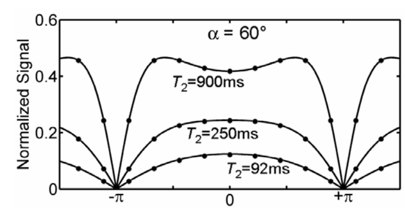

Fig. 5: Apparent relaxation time T_1^* (theory and simulation)

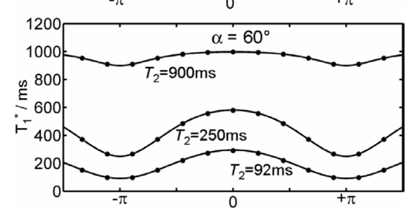

Fig. 6: Transient signal: theory (dotted) and experiment (gray - α/2 prep, black - ideal prep)