Effects of T_2 Decay on the Point-Spread Function for 3D Radial FID Sampling

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Introduction

3D radial free-induction-decay (FID) sampling schemes can be used for ultrashort echo-time imaging (UTE) [1]. With echo times in the order of 100 µs and below, this technique enables the detection of species with transverse relaxation times T_2 in the submillisecond range. Since typical sampling window durations T_{AQ} are in the order of several hundred microseconds, effects of T_2 decay during readout have to be considered. To this end, the single-voxel point-spread function (PSF) resulting from a 3D radial FID sampling pattern is calculated. For this center-out sampling scheme, T_2 decay during readout leads to centric signal decay in k space, whose effect on the 3D PSF is discussed and compared to a radial 2D and a 1D scenario.

Methods and Results

Figure 1 depicts a typical 3D UTE sequence. After a non-selective excitation pulse and a coil-dependent switching time in the order of 100 µs, the readout gradient is ramped up, and the acquisition of the FID is started. Thus, k space mapping begins at k = 0 and proceeds outwards radially. In order to ensure isotropic k-space coverage [2], projections are arranged in the 3D fashion depicted in Fig. 1.

In the **absence of signal decay**, the PSF $P_{3D}(r)$ for this sampling scheme can be obtained from the 3D Fourier transform of a homogeneous sphere of radius k_{max}, described by $S(k) = \Pi(k/k_{max})$ in k space [3]. Table 1 compares the analytic result with the 2D radial and 1D (cartesian) case. Amplitudes are normalized to unity. The left diagram in Fig. 2 plots these PSFs in units of the pixel size π/k_{max}. The curves differ in shape, with the 3D function having a broader main peak, but less side-lobe amplitude than its 2D ("Jinc" [4]) and 1D (Sinc) equi-valents. As a measure of linewidth, the 2nd column of Tab. 1 lists full width at half maximum values (FWHM).

A **centric exponential decay** in k space under the simplifying assumption of a constant readout gradient G, i.e. the linear relation $k(t) = \gamma Gt$, is described by $S_{dec}(k(t)) = \exp(-t/T_2) = \exp(-kT_{AQ}/k_{max}T_2)$. Hereby, k is the radial component, that reaches its maximum value after a sampling duration T_{AQ}, so that $k_{max} = \gamma GT_{AQ}$. The 3D Fourier transform of $S_{dec}(k)$ is the Lorentzian-type image space "blurring" function $P_{dec,3D}(r)$. It is given together with the corresponding 2D and 1D functions in column 3 of Tab. 1. The center diagram of Fig. 2 shows normalized plots for a T_{AQ}/T_2 ratio of 4. Clearly, the 3D function has the smallest linewidth. FWHM values are listed in column 4 of Tab. 1.

The **total point-spread function** $P_{tot,3D}(r)$ is obtained by a 3D convolution of the two functions: $P_{tot,3D}(r) = P_{3D}(r) *** P_{dec,3D}(r)$. The rightmost diagram in Fig. 2 compares this single voxel PSF with its 2D and 1D analogons.

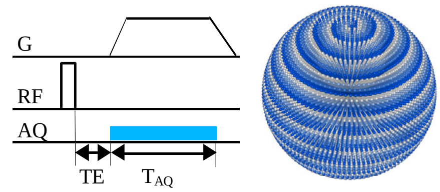

Figure 1: 3D ultrashort TE sequence applying a non-selective excitation pulse and FID sampling (left). 3D coverage of k space using an isotropic arrangement of radial projections (right). Different colors indicate interleaved subsets of projections.

	PSF without decay $P(r)$	**FWHM [pixels]**	**Blurring Function $P_{dec}(r)$**	**FWHM [pixels]**
3D radial	$3\dfrac{\sin(k_{max}r) - k_{max}r\cos(k_{max}r)}{(k_{max}r)^3}$	$\Delta r \approx 1.59$	$\dfrac{(k_{max}T_2/T_{AQ})^3}{\left(1+(r\,k_{max}T_2/T_{AQ})^2\right)^2}$	$\Delta r \approx 0.41\,T_2/T_{AQ}$
2D radial	$2\dfrac{J_1(k_{max}r)}{k_{max}r}$	$\Delta r \approx 1.41$	$\dfrac{(k_{max}T_2/T_{AQ})^2}{\left(1+(r\,k_{max}T_2/T_{AQ})^2\right)^{3/2}}$	$\Delta r \approx 0.49\,T_2/T_{AQ}$
1D	$\dfrac{\sin(k_{max}r)}{k_{max}r}$	$\Delta r \approx 1.20$	$\dfrac{k_{max}T_2/T_{AQ}}{1+(r\,k_{max}T_2/T_{AQ})^2}$	$\Delta r \approx 0.64\,T_2/T_{AQ}$

Table 1: 3D radial PSF in the absence of decay and blurring function in comparison with their 2D radial and 1D analogons. Full width at half maximum linewidths (FWHM) Δr are given in terms of pixels (π/k_{max}). J_1 is the first-order Bessel function of the first kind.

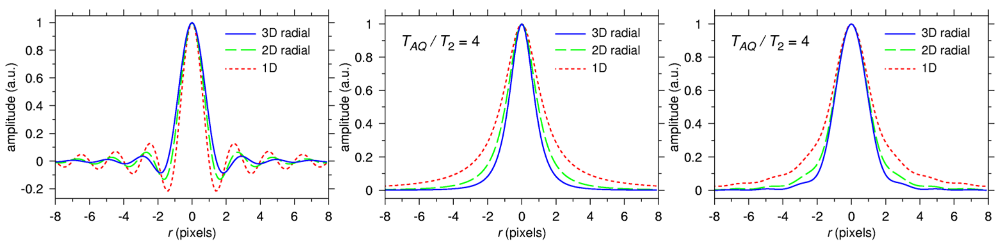

Figure 2: PSF in the absence of T_2 decay (left), blurring function for $T_{AQ} / T_2 = 4$ (center), and convolution of both (right) for different sampling schemes. The convolved function is the PSF in case of centric T_2 decay.

Discussion

From the calculation of $P(r)$ and $P_{dec}(r)$ one finds that the 3D PSF has a larger "intrinsic" linewidth, but is affected less by T_2 broadening, i.e. blurring, than the 2D and 1D PSFs. Therefore, a crossover value of T_{AQ}/T_2 exists, after which the total 3D PSF has a smaller linewidth than the 2D and 1D functions. The left diagram of Fig. 3 plots the FWHM versus T_{AQ}/T_2. For T_{AQ}/T_2 roughly larger than 3, the 3D PSF is narrower than its 2D and 1D equivalents.

On the other hand, the numerators of the blurring functions given in Tab. 1 show that the PSF amplitudes decrease with different powers of T_{AQ} / T_2 for the three sampling schemes. This effect leads to a larger amplitude loss with decreasing T_2 for the 3D case, as plotted in the right diagram of Fig. 3. The underlying reason is the centric nature of signal decay in k space in combination with the k^2 weighting of the 3D radial samples [1]: undecayed signal at low k values is thus underweighted with respect to already decayed signal picked up at higher k values, resulting in an increased loss in PSF amplitude compared to 2D radial or 1D sampling. In an image, however, blurring can cause signal intensity from neighboring pixels to overlap, and thus reduce the effect of signal loss found for the single-voxel PSF.

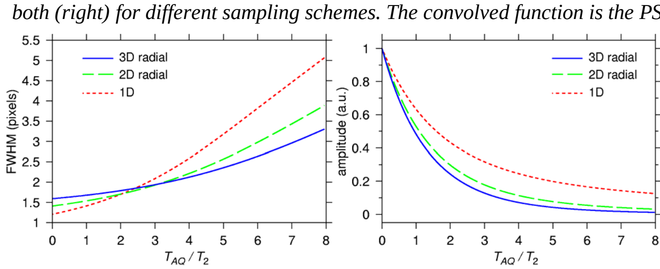

Figure 3: T_2-decay induced line-broadening of the total PSF for different sampling schemes (left). The 3D PSF is less affected by blurring than the 2D and 1D PSFs. On the other hand, the PSF amplitude decreases more strongly with shorter T_2 for the 3D case (right).

Conclusion

Calculation of the single-voxel PSF for 3D radial FID sampling under the influence of T_2 decay exhibits reduced blurring compared to a 2D radial FID or a 1D scenario. On the other hand, the 3D PSF is subject to a larger amplitude loss with shortening T_2, the effects of which on the image depend on the spatial signal distribution, but can lead to lower SNR for short-T_2 signal acquired with the 3D radial FID technique.

References

[1] Glover GH *et al.*, J. Mag. Res. **2**, 47-52 (1992). [2] Wong S *et al.*, Mag. Res. Med. **32**, 778-784 (1994). [3] Bracewell RN, The Fourier Transform and its Applications. New York: McGraw-Hill, Inc., 1986. [4] Lauzon ML *et al.*, Mag. Res. Med. **36**, 940-949 (1996).