The Effect of Boundary Conditions on Dispersive and Non-Dispersive Systems using Magnetic Resonance Elastography and Finite Element Analysis

S. I. Ringleb[1], Q. Chen[1], A. Manduca[2], K-N. An[1], R. L. Ehman[2]

[1]Biomechanics Laboratory, Division of Orthopedic Research, Mayo Clinic, Rochester, MN, United States, [2]Department of Radiology, Mayo Clinic, Rochester, MN, United States

Introduction

Inversion methods used to process data from Magnetic Resonance Elastography (MRE) that are based on the equations of motion calculate the underlying shear wavelength from complex wave fields, assuming an infinite, homogenous material (1). In a finite solid, the boundary conditions can interfere with shear wave propagation. Moreover, for a homogenous elastic solid beam undergoing flexural motion, the shear wave speed changes with excitation frequencies (i.e., a dispersive system, in contrast to constant wave speed with various frequencies), and applied axial tension (T) has a pre-stress stiffening effect that causes the shear wavelength to be longer than in otherwise unloaded elastic material (2): $\lambda=2\pi\{-\xi+(\xi^2+\omega^2/a^2)^{1/2}\}^{-1/2}$, where $\xi=T/2EI$, $\alpha=EI/\rho A$, E is Young's modulus of the material, I is the moment of inertia, ρ is the density and A is the cross-sectional area. In the present study, a finite boundary homogenous isotropic elastic axisymmetric FE model was developed to examine the influences of shear modulus (G), density (ρ), excitation frequency (f) and tension (T) on shear wavelength (λ) and was compared to MRE measurements on soft tissue mimicking phantoms.

Methods

A bar-shape polyvinyl-siloxane material (Wirosil, Bremer Goldschlägerei Wilh. Herbst GmbH & Co., Bremen, Germany) was supported at both ends. The bar was connected to a pulley system through which a varying amount of tension (0-210 kPa) was applied uni-axially to the phantom. MRE images at each tension level at given frequencies from 100-450 Hz were collected. The measurements were repeated when the bottom of the phantom was constrained. The phantom bar was modeled as a rectangular prism plane strain model in Abaqus (HKS, Warwick, RI) using 2mm x 2mm four-node bilinear elements (CPE4) constrained in the vertical direction only at the left-bottom vertex and the right-bottom vertex (Fig. 1). Various amounts of axial tension were modeled as distributed surface pressure along the two lateral edges. Sinusoidal excitation was prescribed in the center of the model. Transient dynamic analysis was used to characterize the propagating shear waves in the model. The same simulation method with the entire bottom of the prism constrained in the vertical direction was also performed.

Results

The shear wavelengths λ_m measured at various frequencies with no tension were plotted in a log-log plot (Fig. 2). Linear regression shows a slope of -0.69, indicating a power law relationship between the shear wavelength and the excitation frequency of order -0.69 in the tests. Consistently, the shear wavelength estimated from the FE model shows a similar relationship between the shear wavelength and the frequency, with a slope of -0.59. The shear wavelength at different tension levels at a given frequency increases with tension with a slope of 0.220mm/KPa at 100 Hz, 0.087mm/KPa at 150 Hz, and 0.057mm/KPa at 100 Hz when evaluated with linear regressions. Correspondingly, the FE model also demonstrates that the shear wavelength increases with the axial tension in same slopes (Fig 3). By contrast, when the entire bottom of the phantom was constrained in the vertical direction, both the MRE experiment and the FE simulation show that the propagating shear wavelength does not significantly change with axial tension, and the log-log plot indicates a slope of -1, indicating an inverse proportional relationship, and therefore a non-dispersive system.

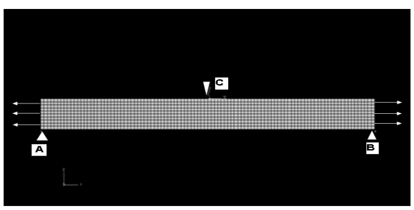

Fig. 1 2D plane strain FE model simulating the flexural wave in the phantom

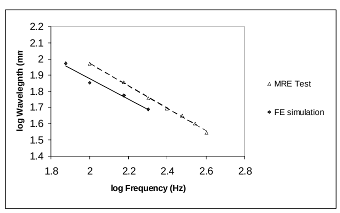

Fig 2. Log-log plots of shear wavelength measured at various frequencies at tension T=0

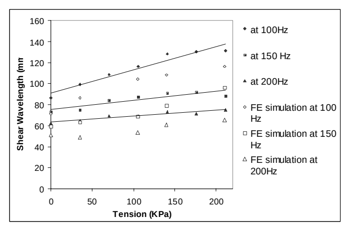

Fig 3. Shear wavelength in the phantom at different tension levels in MRE test and the FE simulation 100, 150 and 200 Hz and at different axial tension levels

Discussion

The finite element method offers a feasible and powerful method to quantitatively study the relationship between the shear wavelength and various parameters of the media. Finite element based elasticity reconstruction has been used with a squared error minimization formulation (3) or energy cost minimization formulation (4), both of which compared the actual displacement to the displacement in a finite element mesh. In the FE models in the present study, shear wavelength is the primary parameter studied. The shear wavelengths obtained from the FE model (λ_f) agree well with those estimated from the MRE measurements (λ_m), suggesting MRE is a valid and reliable method to estimate the homogenous material stiffness.

Results in both the MRE experiment and the FE modeling show that depending on the boundary conditions, the system may be dispersive or non-dispersive, and the pre-tension effect on the shear wavelength may also vary. Particularly, when the bottom was supported in the vertical direction only at the ends, the shear wave propagation can be described by a dispersive flexural wave system. This information must be considered when applying MRE to *in vivo* soft tissue. For example, muscle is suspended between tendons, therefore, it is reasonable to infer that it could be somewhat dispersive, and the shear wavelength in the muscle may change with applied tension.

References

(1) Manduca, A., et al., 2001. Medical Image Analysis, 5, 237-254.
(2) Graff, K. F., 1991. Wave Motion in Elastic Solids. Oxford, UK, pp. 278-279.
(3) Van Houten, E., et al., 2000. Medical Physics 27, 101-107.
(4) Harrigan, T. P., et al., 2004. Journal of Biomechanics 27, 1215-122.

Acknowledgements

This study is supported by NIH grants EB00812 and EB01981 and CA91959. We thank Thomas C. Hulshizer for his technical assistance.