Equiripple RF pulse design using inverse scattering theory

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Introduction

The Shinnar-Le Roux (SLR) algorithm has been used to produce so-called *equiripple* slice-selective RF pulses [1]. The magnetization profile of such a pulse (obtained via the Bloch equation) has an error which is uniformly minimized outside of a user-specified *transition region* which surrounds the selection band. Recently [2], it has been shown that analogous equiripple pulses can be designed using the inverse scattering theory (IST) method of pulse design. In both cases, the design parameters are: flip angle, slice thickness, in-slice ripple, out-of-slice ripple, transition width, and rephasing time. Here we compare the SLR (linear phase) and IST methods of equiripple pulse design using a computer simulation. Specifically, the out-of-slice ripple is computed as a function of the in-slice ripple for various transition widths and rephasing times. For simplicity, the slice thickness and flip angle are fixed at 1 KHz and 90° throughout, and the magnetization is normalized to be a unit vector.

Theory

The first step in both the SLR and IST algorithms is to generate a trigonometric polynomial P using the Remez algorithm such that P is close to 1 in-slice and close to 0 out-of-slice (see **Figure 1**). The parameters for the Remez algorithm are the degree of the polynomial, the transition width, the in-slice ripple (δ_1), and the out-of-slice ripple (δ_2). For the SLR algorithm this polynomial defines the so-called *Cayley-Klein* polynomials a and b which satisfy $|a|^2+|b|^2=1$ (for details see [1]). These polynomials are the input to the SLR recursive algorithm which computes the RF pulse with the appropriate equiripple magnetization profile. Similarly, the IST method uses P to define the so-called *reflection coefficient* (for details, see [2]), which is then used to produce an RF pulse with an equirriple magnetization profile. In both cases the degree of the polynomial directly determines the required rephasing time for the pulse. The in-slice error in the longitudinal magnetization and the out-of-slice error in the transverse magnetization are determined by the Remez ripple parameters δ_1 and δ_2., and the formulas are given in **Table 1**. Again we refer the reader to [1] and [2] for a detailed discussion of these parameter relations.

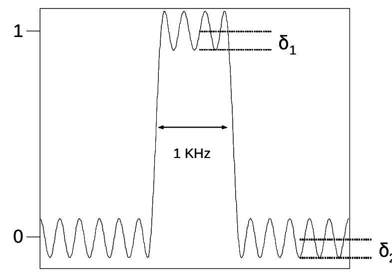

Figure 1. Equiripple polynomial from the Remez algorithm.

Methods

Matlab's filter design toolbox was used to generate equiripple polynomials for various transition widths and rephasing times. The slice thickness was fixed at 1 KHz, and the in-slice ripple was varied. The out-of-slice error was measured numerically, and the formulas in **Table 1** were then used to compute the out-of-slice error in the magnitude of the transverse magnetization as a funciton of the in-slice error in the longitudinal magnetization for both the SLR (linear phase) and IST methods.

	in-slice error	out-of-slice error
SLR	$2\delta_1+\delta_1^2$	$\sqrt{2}\delta_2\sqrt{1-\frac{1}{2}\delta_2^2}$
IST	$\frac{\delta_1-\frac{1}{2}\delta_1^2}{1-\delta_1+\frac{1}{2}\delta_1^2}$	$\frac{2\delta_2}{1+\delta_2^2}$

Table 1. Formulas for the in-slice error in the longitudinal magnetization and the out-of-slice error in the magnitude of the transverse magnetization for the SLR (linear phase) and IST equiripple 90° pulse design techniques. Here, δ_1 and δ_2 are the ripple amplitude parameters for the Remez algorithm as described in the *Methods* section.

Results and discussion

For most parameters, the IST method performed better in the sense that, for a fixed in-slice error, the out-of-slice error was smaller for IST (**Figure 2**).

Note that the IST pulses have a somewhat longer total duration than the corresponding SLR pulses (see **Figure 3**). However, because the rephasing times are the same, the magnetization should experience the same T2 decay during the pulse, so in this sense the comparison is meaningful (see [2]).

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References

1. Pauly J, et al, *IEEE Tr Medical Imaging*, **10**: 53-65 (1991);
2. Magland J, et al, *J Magn Reson*, 1-26, in-press (2004).

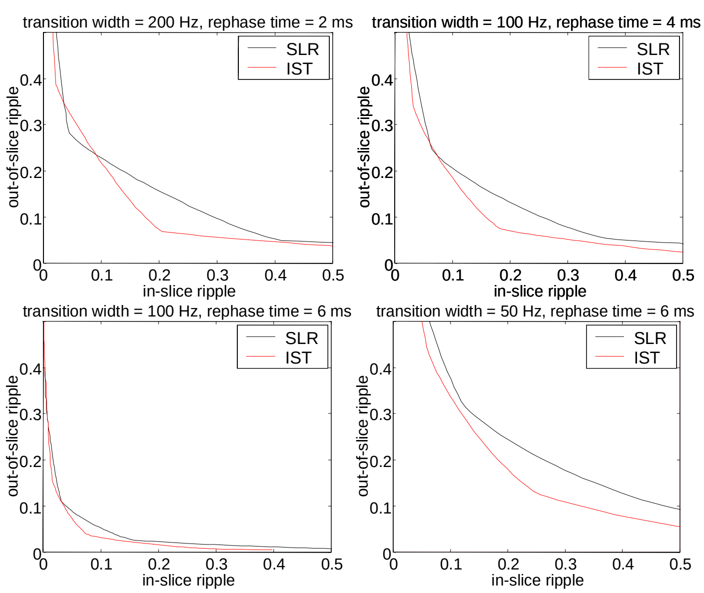

Figure 2. Out-of-slice ripple as a function of in-slice ripple for equiripple 90° SLR and IST pulses with various transition widths and rephasing times.

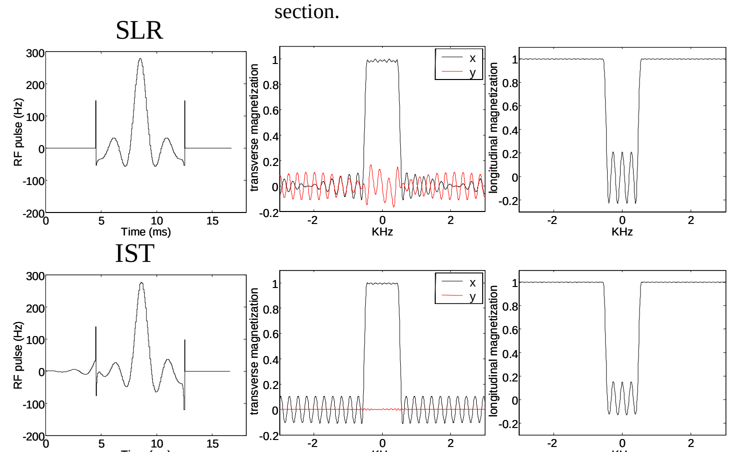

Figure 3. Equiripple 90° SLR and IST pulses with corresponding magnetization profiles for a transition width of 100 Hz, out-of-slice ripple of 0.1, and a rephasing time of 4 milliseconds. The pulses were generated using the algorithms in [1] and [2], and the magnetization profiles were obtained by numerically solving the Block equation. Notice that the IST pulse provides better control of the in-slice phase of the magnetization.