Method for Susceptibility Calculation in Multiple Source Object Distribution with Arbitrary Susceptibilities: A Preliminary Report

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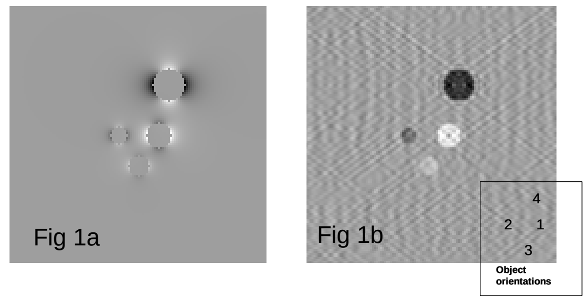
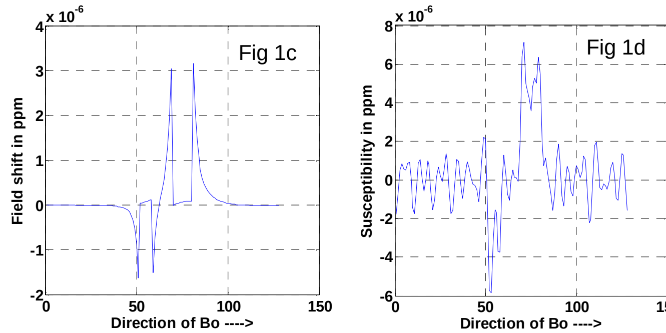
Introduction: Quantitative estimation of magnetic susceptibility of body tissues using MRI has important clinical implications. For example, the ability to quantify susceptibility of a hemorrhagic lesion can help differentiate it as acute or chronic and can help determine its size. Measuring oxygen saturation may also fall to quantitative MR susceptometry. To date, a number of methods have been proposed for susceptometry using MR (1-4). Some of them are based on polynomial fitting of the background fields (2), some on solving for it as an inverse problem in the spatial domain using singular value decomposition (1). These methods are either quite time consuming or are inefficient for closely placed objects that result in complex phase behavior. A number of investigators have looked at a Fourier approach to the forward problem of starting with a given object and predicting its field behavior (5-7). We extend this concept by looking at the inverse problem to find the susceptibility of the source field for a given shape as measured in the MR image itself. We apply the method to a system of closely placed multiple objects with arbitrary susceptibility values and then discuss its properties and applicability to practical situations in MR.

Theory and Methods: Susceptibility quantification from magnetic field data is a problem with no unique solution, as there can be more than one source of susceptibility distributions that can produce the same field. MR imaging offers two interesting constraints to this problem: first the object size and geometry can be determined and second the measured magnetic field is along the z direction. These facts, in our case, ensure a unique solution. Magnetic susceptibility is a material property of a substance which tells us how much the substance would get magnetized, and in what direction, when kept in a strong external magnetic field. When a set of point objects (point dipoles), with different susceptibilities, are placed in an external magnetic field $\mathbf{B_0}$, the $\mathbf{B(r)}$ field experienced by any object at a position $\mathbf{r}$ is a sum of the fields from a) its own magnetization $\mathbf{M}$, and b) the magnetic field at $\mathbf{r}$ due to the magnetizations of the surrounding objects at $\mathbf{r'}$ and the applied background field $\mathbf{B_0}$ itself (assuming uniform B_0. The resultant $\mathbf{B(r)}$ field is given by:



$$B(r) = B_0 + \frac{\mu_0}{4\pi}\int_{V'}\left\{\frac{3M(r').(r-r')}{|r-r'|^5}(r-r') - \frac{M(r')}{|r-r'|^3}\right\}dV'$$



Taking into consideration the effect of the Lorentzian sphere term and the **z** directional dependence of the applied magnetization $\mathbf{B_0}$ and hence $\mathbf{M(r)}$, the Fourier transformation of the above expression gives (5-7):

$$(\chi(r)) = FT^{-1}\left[FT\left[\frac{\varphi(r)}{-\gamma B_0 TE}\right]\cdot\frac{3\left(K_x^2+K_y^2+K_z^2\right)}{K_x^2+K_y^2-2K_z^2}\right]$$

where $\varphi(\mathbf{r}) = -\gamma(\Delta B_{obj})TE$. In the above expression, $\varphi(\mathbf{r})/(-\gamma B_0 TE)$ represents the total $\Delta B(\mathbf{r})$ shift from all objects. Since $\Delta B = \Delta B_{obj\ 1} + \Delta B_{nobj\ 2} + \Delta B_{obj\ 3}$ …, a linear sum of field shifts due to individual objects, we can invoke the linearity of the Fourier transform and expect that the theory will lead to the de-convolution of the susceptibility for multiple objects. Further, the shift theorem of the Fourier transform, states that a shift in the image leads to a linear phase shift in k-space. Since the inverse filter is a real filter in k-space, it can be used to extract the susceptibility independent of object position.

Results: A series of four spheres is simulated in a 128 x 128 x 128 matrix and the resulting field deviation map is shown in figure 1a, along with a profile plot across objects 1 & 2 (see fig 1b) in figure 1c. With such a field distribution, the phase from one sphere interferes with that from another, making a simple polynomial fit to the total field extremely difficult. The susceptibilities and the diameters of the spheres are -8, 2, -3, 5 (in ppm) and 16, 10, 8, 12 pixels, respectively. The susceptibility map obtained by the k-space filtering method is shown in figure 1b along with a profile across the objects 1 & 2 in figure 1d. The result was low-pass filtered to get a smoother χ profile. The resultant mean and standard deviations of χ are shown in Table 1. The error in χ and the standard deviation vary with both the object size and the χ value. In general, the error increases as the diameter/FOV ratio increases. This can be seen from the values obtained from the nearly point object simulation (with the radius of the sphere set to ½ pixel) as given in Table 2.

Object	Simulated χ (in ppm)	measured χ (ppm)	Standard Deviation
1 (d=12)	5	5.16	0.97
2 (d=8)	-3	-3.45	0.94
3 (d=10)	2	2.16	0.58
4 (d=16)	-8	-8.13	0.98
Background	0	0.04	0.60

Table 1

Object	Simulated χ	Calculated χ	% Error in χ
1	5	5.222	4
2	-3	-3.042	1.4
3	2	1.9652	-1.7
4	-8	-8.354	4.4

Table 2

Discussion and Conclusion: To explain the changes in χ as the diameter-to-FOV ratio increases, we note that the surface term is no longer negligible. Basically, as the diameter-to-FOV ratio increases, there is little information from the phase outside the object and this leads to the observed error and, we believe, the ringing. The method only relies on the phase information obtained from the MR experiment. So, ideally the phase map has to be free from any geometric distortion effects and from aliasing. This can be achieved by using high bandwidth read out gradients and using short TE to acquire the phase information, respectively. Another potential source for error is the finite resolution of the input phase map. One way to overcome this is to collect high resolution phase images. In summary, we have described a new approach for creating a susceptibility map directly from the acquired k-space data. This approach simplifies the quantification of susceptibility in MR imaging and should make its use more widely available for future clinical studies such as for quantifying χ for small hemorrhages or finding oxygen saturation in small veins, both of which are important for better clinical diagnosis of the patient's status.

References: **1**) Lin Li, Leigh S, MRM 51:1077 (2004). **2**) Weisskoff RM, Kiihne S. MRM 24: 375 (1992). **3**) Holt RW, Diaz PJ, Duerk JL, Bellon EM. JMRI, 4:809 (1994). **4**) Wang ZJ, Li S, Haselgrove JC, JMR, 140:477 (1999). **5**) Deville G, M. Bernier, Delrieux JM Phys. Rev. B. 19(11):5666 (1979). **6**) Marques JP, Bowtell R, Proc. Intl. Soc. Mag. Reson. Med. 11:1020. **7**) Salomir R, Senneville BD, Moonen Chrit TW, Concepts in Magnetic resonance Part B. 19B(1): 26 (2003).