MR Image Resolution Enhancement through Wavelets

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Introduction

Spatial resolution is an important characteristic of MR images. By definition, spatial resolution is the smallest distance between two different objects, or two different features of an object, at which their images are distinct. In MRI, there is an inverse relationship between spatial and temporal resolution during image acquisition, even when multi-receiver systems are used. Therefore, it is extremely useful to improve image resolution by post-processing methods. The superresolution technique described here attempts to address the problem of in-plane resolution via wavelet-based variational methods.

Method

Measured *k*-space data can be viewed as the Fourier coefficients of a function in $L^2(Q)$, where $Q=[0,1]^2$. Since one can only measure finitely many Fourier coefficients, the data only determines a trigonometric polynomial, and the discrete Fourier transform (DFT) reconstructs a highly 'voxelized' version of this polynomial. Our variational approach seeks to reconstruct an image which is of higher resolution than this (low resolution) voxelized trigonometric polynomial, such that the new image has *k*-space data which agrees closely with the low resolution image's *k*-space data, and such that the new image has a small norm in a certain smoothness space. The space BV (bounded variation) is a natural and common space for such image analysis. Our variational approach based on wavelets is motivated by the BV image denoising techniques of Rudin and Osher.[1] Given an image $g\in L^2(Q)$, and a known blurring operator T, we seek to reconstruct *f* which minimizes a discrete version of

$$\|Tf-g\|^2_{L^2(Q)}+\lambda\|f\|_{B^1_1} \qquad (1)$$

where $\|\cdot\|_{B^1_1}$ is a Besov space norm capable of reducing oscillations such as the Gibbs effect, which has been shown to be a good substitute for BV. The variational functional (1) is minimized via the iterative thresholding algorithm of Daubechies, Defrise, and DeMol.[2]

This superresolution technique was tested on low resolution phantom data and compared to the actual high resolution data. All data were acquired using a Siemens Allegra 3.0 Tesla head-only scanner.

Results

The superresolution algorithm was tested on a cylindrical phantom (Figure 1) consisting of D_2O filled NMR tubes surrounded by water and held in place by styrofoam at both ends.

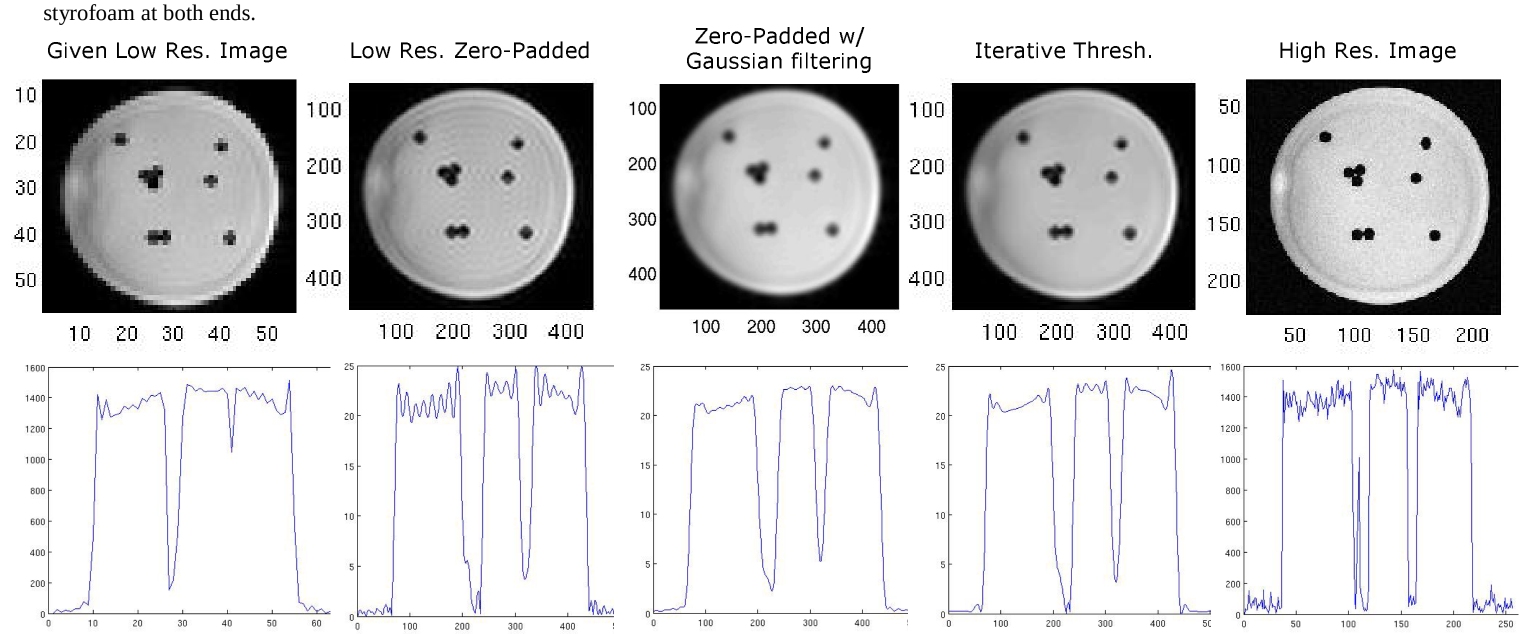

Figure 1:

The top array of images allows for comparison of our superresolved image of one slice of the NMR tube phantom (imaging parameters: 128 FOV, TR=2500ms, TE=15 ms, 2.5mm slice thickness). The low resolution data (64x64 matrix) was first interpolated by zero-filling. To this, the wavelet based variational minimization was applied. The superresolved image after iterative thresholding is shown fourth from the left. When compared to the low resolution data it can be seen that objects are more distinguishable especially in the region of the bundle of three NMR tubes. It also has reduced Gibb's ringing and the 'voxelizing' effect of the DFT. Compared to the zero-padded version with Gaussian filtering, our superresolved image is more distinct (i.e. less blurry). It also shows good agreement with the high resolution (256x256 matrix) image.

The bottom array of graphs shows intensity values for a chosen vertical line through the image. The presence of the second dip in the graph is more pronounced in the high resolution image than in the low resolution image. Zero-padded, zero-padded w/Gaussian filtering, and iterative thresholding images are very similar in regard to the dips, but exhibit different degrees of smoothness at the tops of the peaks.

References:

1. Rudin et-al. *Physica D*. 60;259-268, 1992.
2. Daubechies et-al. *Communications on Pure and Applied Mathematics.* 57(11);1413-1457, 2004.