Spatial-Temporal Regularization Framework for Dynamic MRI Series Reconstruction

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Introduction: Reducing amount of MRI data required for image reconstruction is an important strategy to increase spatial/temporal resolution and volume coverage in many dynamic MRI applications. A variety of different methods for acquisition and reconstruction of reduced datasets were proposed [1-4]. Most of them rely on explicit [1] or implicit [2-4] temporal filtering and special acquisition strategies to optimally use the available temporal bandwidth. Each individual method has its own advantages and limitations defined by *a priori* assumptions about temporal dynamics of the object and filters used for reconstruction. If assumptions are violated, artifacts arise that may lead to incorrect results [5].

In this paper, we describe a general framework for reconstruction of dynamic MRI series from reduced data based on Tikhonov regularization [6]. The framework is flexible to accommodate many linear and nonlinear spatial and temporal filters in the form of Tikhonov regularization terms. Additionally, we present two new methods for reconstruction of dynamic MRI data based on the described framework.

Theory: We consider reconstruction of dynamic MRI data as the following optimization problem:

$$\arg\min_{\bar{\mathbf{f}}}\left(\left\|\bar{\mathbf{E}}\bar{\mathbf{f}}-\bar{\mathbf{m}}\right\|_2^2+\sum_{i=1}^{N_S}\lambda_i^2 S_i\left(\bar{\mathbf{f}}\right)+\sum_{i=1}^{N_T}\eta_i^2 T_i\left(\bar{\mathbf{f}}\right)\right), \quad (1)$$

where $\bar{\mathbf{E}}\equiv\mathbf{I}_n\otimes\mathbf{E}$, $\otimes$ is the Kronecker product, $\mathbf{I}_n$ is the $n\times n$ identity matrix, n is the number of time frames, $\mathbf{E}$ is the encoding matrix composed of Fourier terms, or of coil sensitivities weighted Fourier terms for parallel MRI (P-MRI). Vectors $\bar{\mathbf{f}}$ and $\bar{\mathbf{m}}$ contain solution and data points, respectively. $S_i\left(\bar{\mathbf{f}}\right)$ and $T_i\left(\bar{\mathbf{f}}\right)$ are regularizing terms having effects of spatial and temporal filtering, respectively, and λ_i,η_i are regularization parameters to balance goodness of data fit and filtering strengths. The formulation was also studied in electrocardiography [8].

When the regularizing terms have a form of L_2 norm, solution to (1) could be obtained as the result of least squares estimation:

$$\bar{\mathbf{f}}=\left(\bar{\mathbf{E}}^{\mathbf{H}}\bar{\mathbf{\Psi}}^{-1}\bar{\mathbf{E}}+\sum_{i=1}^{N_S}\lambda_i^2\mathbf{S}_i^{\mathbf{H}}\mathbf{S}_i+\sum_{i=1}^{N_T}\eta_i^2\mathbf{T}_i^{\mathbf{H}}\mathbf{T}_i\right)^{-1}\bar{\mathbf{E}}^{\mathbf{H}}\bar{\mathbf{\Psi}}^{-1}\bar{\mathbf{m}}, \quad (2)$$

where $\bar{\mathbf{\Psi}}$ is chosen to account for noise levels and correlations in coil channels for SNR-optimal reconstruction, and $\mathbf{S}_i$ and $\mathbf{T}_i$ are spatial and temporal filtering matrices, correspondingly. One example of a spatial filter is the 0^{th} order filter given by $\mathbf{S}_i=\mathbf{I}$, and its adaptive P-MRI version [7]. In our methods, we use the 1^{st} order temporal filter:

$$T\left(\bar{\mathbf{f}}\right)=\left\|\mathbf{L}\bar{\mathbf{f}}\right\|_2^2, \quad (3)$$

where matrix $\mathbf{L}$ is the first derivative operator in time dimension. The filter design could be based on *a priori* assumptions on temporal dynamics of the object. Additionally, filtering effects could be tuned using adaptive regularization parameters.

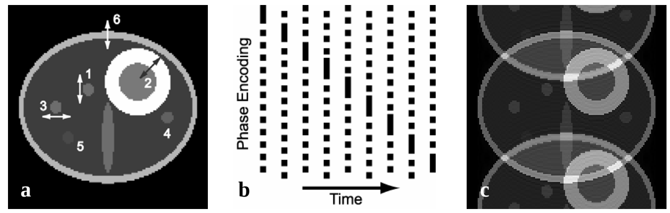

Figure 1. Digital phantom (40 frames, each frame is 128-by-128) to simulate heart and respiratory motions. **a:** Phantom with dynamic structures. **b:** *k*-Space sampling (R=1.83). **c:** Direct Fourier inversion of reduced data.

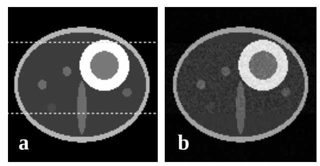

Figure 2. Reconstruction in absence of respiratory motion (without motion #6 in Fig. 1a). **a:** Reconstructed image (RMSE=2e-4). The central part was assumed dynamic (η=0) and the rest is static (η=500). **b:** Image reconstructed from noisy data.

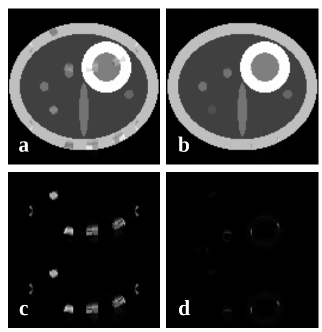

Figure 3. Reconstruction of phantom with simulated respiratory motion. **a:** Nonadaptive reconstruction (RMSE=0.117) **b:** Adaptive reconstruction (RMSE=0.017). **c,d:** Error images corresponding to *(a)* and *(b)*, respectively. Nonadaptive method, Noquist-type technique was not able to resolve aliased pixels where temporal spectra were overlapped. Adaptive choice of η leads to acceptable results.

Methods: Next, we describe two new methods for reconstruction of dynamic series based on the 1^{st} order regularization (3). In all examples, minimization of (2) was done by iterative Conjugate Gradient algorithm. Sampling pattern and numerical phantom simulating dynamics of cardiac MRI were identical to the ones used in [3] (Fig. 1).

The first, nonadaptive method assumes that certain object parts are static while the rest of the object is characterized by nonzero temporal dynamics. The assumption could be enforced by setting regularization parameter to a large value in static areas (η=500, large filtering), and to zero in the rest of the object (η=0, no filtering). The approach is similar to Noquist technique [3] developed to reconstruct time frames by direct matrix inversion, which assumes static area solution is common for all time frames. The results are shown in Fig. 2.

The second, adaptive method assumes that temporal dynamics of object is known, for example, from training scans or employing navigator information. We solve (1) using filter (3) with regularization parameter varying gradually from minimum to maximum values in accordance with $\left\|\mathbf{L}\bar{\mathbf{f}}_{train}\right\|_2$, where $\bar{\mathbf{f}}_{train}$ is training data. To test the adaptive method, we added simulated respiratory motion to the digital object (Fig. 3). Such motion results in aliased pixels with overlapping temporal spectra (Fig. 1c). The nonadaptive reconstruction was implemented setting η to 0 in dynamic and to 500 in static areas. The results are shown in Fig. 3.

Discussion and Conclusions: We described a general framework for reconstruction of dynamic MRI series from undersampled data using Tikhonov formulation. The framework is flexible to accommodate any number of temporal and spatial filters for regularized solution. Many existing methods [1-4, 6] have an equivalent formulation inside the proposed framework. It allows combining them to fulfill needs of a particular MRI application. The filtering effects of individual methods could be balanced by adjusting regularization parameters. One way to find the optimal set of regularization parameters is to use L-surface algorithm [8]. The framework also allows nonlinear filters such as total variation filter. Using nonlinear filters [4] may help recover rapid dynamic changes that are excessively penalized by L_2 norm (3), and is an interesting area of the future research.

Next, we proposed two new methods for reconstruction of undersampled dynamic MRI data based on the 1^{st} order temporal regularization. The first, nonadaptive method produces high quality reconstruction of time series when Noquist-type assumption is met. The second, adaptive technique provided good results even in the presence of aliased pixels with overlapping temporal spectra due to simulated respiratory motion. Such overlapping is met in many studies due to patient respiratory motion, and is a source of image artifacts in a number of existing methods. The efficiency of the adaptive method comes from incorporating the knowledge about the object's temporal dynamics into reconstruction. In practice, such knowledge could be obtained by means of separate training scans [2].

Acknowledgments:

We are thankful to Dr. Ed DiBella, Dr. Dennis Parker, and Dr. Rob MacLeod for helpful discussions.

References:

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