Under-Sampled Dynamic Data Reconstruction Based on Object Motion Estimation

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Introduction. A reconstruction method for under-sampled dynamic data is presented. Currently, there exists methods like UNFOLD [1] and k-t BLAST [2], which employ the properties of the pixel's intensity changes for the image reconstruction. In contrast, we estimate the motion of subdivisions of the object (let's call them elements) in a continuous space. The main advantage of this framework is that the assumption of motion smoothness is typically valid for object elements whereas pixel's intensity changes are not smooth (think of an edge entering or exiting a pixel). This smoothness allows characterization of the motion with a model that has fewer parameters than the total number of pixels.

The idea is to find a model that fits the image sequence based on a reference frame and a motion definition (Motion Transformation Matrix for each pixel), which allows estimation of all the other frames. This formulation permits a complete reconstruction if the number of degrees of freedom of the model is less than the number of acquired data points. The advantages of this reconstruction algorithm are that, it does not impose temporal frequency restrictions on the pixels; it is applicable to any under-sampling pattern; and it does not require the motion to be restricted to a part of the field of view. Additionally, not only it reconstructs the sequence of images, but it also recovers a good estimation of motion vectors than can be valuable, for example, in analysis of cardiac function from a cine sequence.

Using simulated 2D phantoms we show the potential of the proposed method. However, we have to improve and optimize our current experimental implementation in order to lower the computational load.

Method. Let m_i be the discrete image at time i $(i=1\ldots T)$ of a sequence with T time frames. Let's assume also that each image is N_xN_y pixels. Let $P_i(e)$ be the Motion Transformation Matrix that transforms each pixel of m_0 into m_i according to equation (1), which depends on a vector of parameters e of size n_e. Let us call B_i the k-space data for frame i under-sampled using the pattern S_i with an under-sampling factor Q. Each acquired frame corresponds to (N_xN_y/Q) equations of type (2), where W represents the Fourier Transform. Because there are $N_xN_y+n_e$ unknowns, the sequence becomes completely determined when equation (3) is met.

From a numerical point of view it is convenient to work in the image domain, therefore equation (2) becomes equation (4) where $b_i=W^{-1}B_i$ and $A_i= W^{-1}S_iW$, both known. The problem is then to find e and m_0 to satisfy equation (4). This is done by two nested optimization loops. The inner one finds the optimal m_0 for a given parameter e. This optimization is fast employing a Conjugate Gradient algorithm (we used lsqr from Matlab) and returns the matching error Δb (the difference between the known b and the one obtained from m_0 and e). The outer optimization loop finds the parameter e that minimizes the matching error Δb. Once having the optimal m_0 and e the reconstructed sequence of images is obtained from equation (1).

$$(1)\ m_i = P_i(e)m_0$$
$$(2)\ B_i = S_iWm_i = S_iWP_i(e)m_o$$
$$(3)\ T\cdot(N_xN_y) \ge (N_xN_y + n_e)Q$$
$$(4)\ b_i = W^{-1}B_i = W^{-1}S_iWP_i(e)m_0 = A_iP_i(e)m_0$$

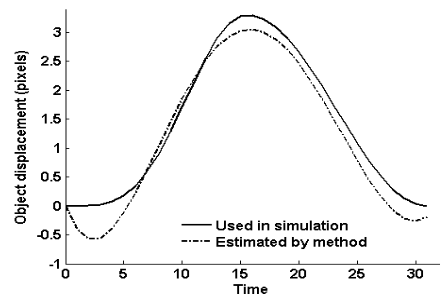

Fig.1.Displacement used in the simulation (solid line) and estimated by the reconstruction for a pixel in the top left circle (dotted line).

Results and discussion. We tested the proposed method with computer simulations. A phantom motion was generated in continuous k-space and sampled to a Cartesian grid. For the first 15 frames the motion followed a sine to the fourth power shape and for the last 15 a sine square shape. The phantom of *64x64* with *32* time frames was under-sampled with a factor *Q=4*. The object movement was approximated by polynomial coefficients (5th order Chebyshev), i.e. the vector e contains the five Chebyshev coefficients for every element. Figure 1 shows the displacement used in the simulation and the estimated by our method. The object movement was estimated using an e of size $n_e = 32x32$, that is, we employed one parameter for every 4 elements. The smooth movement between them was modelled with an interpolating function.

A registration algorithm was employed to obtain an initial estimate of e, lsqr was used to estimate m_0, and Genetic algorithms were used to minimize the error Δb. The simulation required 54 minutes in a typical PC. Fully sampled images for three different frames are shown in the first row of figure 2, and the images obtained with our method for the same frames with an under-sampling factor of 4 are shown in the second row. The reconstructed images have a mean square error of 1.19 % compared to the fully sampled image. The error is mainly due to small differences at the edges of the moving objects. The technique allows a good reconstruction of the object motion.

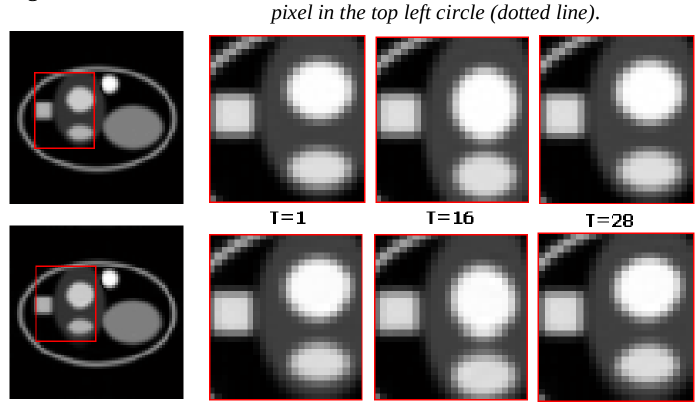

Fig.2.First row: fully sampled images. Second row: our method

Conclusion. We propose a method to reconstruct images from under-sampled dynamic data, employing an estimation of motion of each element of the object. The advantages of this method are that, it does not impose temporal frequency restrictions on the pixels; it does not require the motion to be restricted to a part of the field of view; it is applicable to any under-sampling pattern; it determines object motion as part of the reconstruction process and can be easily extended to 3-D images.

References. [1] Madore et al. MRM 42,5 (1999); [2] Tsao et al. MRM 50,5 (2003).