Reconstruction of MR Images from Sparsely Sampled 3-D k-Space Data

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Introduction

Most magnetic resonance (MR) imaging techniques reconstruct images from fully sampled **k**-space datasets by direct inverse Fourier transform (iFT);[1] however, real-time 3D MR imaging sometimes results in low-resolution [2] or sparsely sampled **k**-space datasets.[3] Specifically, in moving-table contrast-enhanced MR angiography (CE MRA) methods,[3] the patient table is continuously translated along the readout direction (x) while only a portion of the phased-encoded k_y-k_z plane is sampled (with typically acquisition of the centre of k_y-k_z plane favored over the periphery). Because total acquisition time in CE MRA is limited by the duration of the contrast agent arterial first pass,[2] sparsely sampling **k**-space is an interesting and often necessary implementation.[3] Here, we focus on the reconstruction of sparsely sampled hybrid 3-D **k**-space by (a) zero filling (ZF; *c.f.*, Ref [4]) and by (b) projection onto convex sets (POCS).[5] The application of POCS to sparsely sampled **k**-space dataset is a new approach, which differs from its original application on truncated **k**-space.

Method

A moving-table CE-MRA technique [3] was used to acquire a complete raw data set of a quality-control phantom on a clinical 3 T scanner (Signa; General Electric Healthcare, Waukesha, WI). Hybrid **k**-space data (x-k_y-k_z with N_x = 256, N_y = 256 and N_z = 64) were produced by taking the iFT of each readout (x-direction) immediately after acquisition and then placing them into the appropriate location in the hybrid space. Images were reconstructed from the fully sampled hybrid **k**-space (*i.e.*, the true image or $\mathbf{I_0}$) and from simulated sparsely sampled **k**-spaces using both (a) ZF ($\mathbf{I_{ZF}}$) and (b) POCS ($\mathbf{I_{POCS}}$). ZF replaced the missing **k**-space data with zeros, whereas the POCS method forced the missing data to match the acquired data and the phase of the image to match that derived from the fully sampled central zone of **k**-space. The quality of the resulting images was assessed by visual inspection and quantified by calculation of global, as well as local performance errors (PE) – defined as PE = $\sqrt{\sum(\zeta_i - o_i)^2} \Big/ \sqrt{\sum o_i^2}$ where ζ_i and o_i denote pixels from $\mathbf{I_{POCS}}$ (or $\mathbf{I_{ZF}}$) and $\mathbf{I_0}$, respectively.[3] The PE summation was performed over all pixels in the image (global) and over all 3×3 kernels in a local region. The central-zone ratio, $\alpha = N_{\text{central}} / N_{\text{o}}$, was fixed at 25%, where N_{central} and N_{o} are the number of pixels in the central zone and in the complete **k**-space, respectively. The sparsely sampled ratio in the peripheral region, $\beta = N_{\text{null}} / (N_{\text{o}} - N_{\text{central}})$, was varied from 10% to 90%, where N_{null} is the number of missing samples.

Results

POCS successfully reconstructed sparsely sampled 3-D **k**-space data (compare Fig 1a with Fig 1c). Difference images (Figs 1e and 1f) showed that $\mathbf{I_0} - \mathbf{I_{POCS}}$ had generally smaller differences than $\mathbf{I_0} - \mathbf{I_{ZF}}$. Global PE values were smaller in the POCS images. Moreover, as β increased, the error in the POCS image increased more slowly than in the ZF image (Fig 2a). Similar results were also observed in the local PE measures (Figs 2b and 2c) for regions of interest located over high-resolution structures in the phantom (shown in Fig 1a).

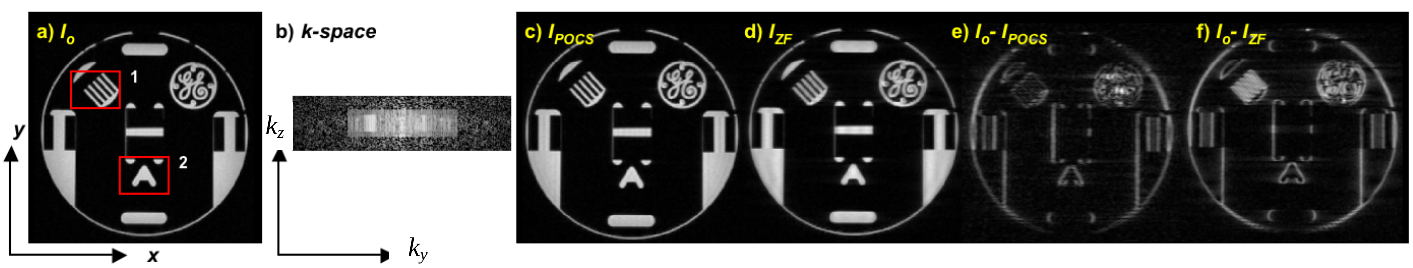

Figure 1: (a) Image reconstructed from complete **k***-space (*$\mathbf{I_0}$*) showing two regions (1,2) used for local PE analysis. (b) Sparsely sampled* **k***-space showing the k_x = 0 plane for α = 25% and β = 50%. (c) POCS (*$\mathbf{I_{POCS}}$*) and (d) ZF (*$\mathbf{I_{ZF}}$*) images and (e,f) difference images.*

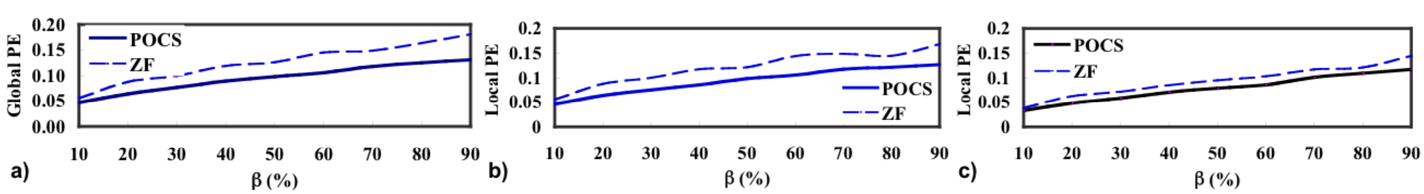

*Figure 2: Performance error (PE) versus sparsely sampled ratio (β) for POCS and ZF images. Shown are (a) the global PE and (b,c) the local PE for the upper (***1***) and lower (***2***) regions defined in Fig 1a.*

Discussion and Conclusions

Compared with ZF, POCS is a better technique for reconstructing sparsely sampled data as it results in good image quality and less global and local error. This is an interesting finding given that POCS was proposed for data extrapolation (*i.e.*, reconstructing truncated **k**-space data),[5] but in this study it has both extrapolative as well as interpolative roles. Further study is required to determine if optimal α and β exist, as well as to investigate time-efficient POCS implementation suitable for use in real-time reconstruction of 3D data sets.

References

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