Image reconstruction of reduced radial data sets using Padé Approximants

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Introduction: Padé approximants (PA) are a particular type of rational fraction approximation to a power series that estimate the fully converged value of the series[1]. MRI data is a Fourier representation of the imaged object and through a simple substitution can be expressed as a power series. Consequently, the PA to the series can be calculated raising the possibility for extrapolating beyond the input data in a manner similar to ARMA modelling[2]. Padé approximants have previously been applied to data acquired with a Cartesian sampling strategy[3]. Here we apply Padé approximants to reduced data sets with a radial trajectory. The truncation artefact which normally results from acquiring insufficient data does not manifest in the Padé reconstruction. These images also have improved spatial definition. This approach could potentially also be applied to single shot spiral k-space trajectories for dynamic studies.

Methods: In order to exploit the extrapolative ability of Padé approximants with MR data, the image must be expressed in power series form. This is achieved by introducing the variable $z=e^{i\Delta k.x}$, where Δk is the sampling interval in k-space; x is the spatial variable. This substitution maps each spatial location, x to a corresponding location, z_x along the unit circle in the complex plane. In the case of a radial acquisition, this means that each line of the sinogram of signal intensity, $I(x)$, versus projection angle, θ, can be expressed in power series form as $I_N(x)=\sum_N C_n(k_x,\theta).z^n$, where $C(k_x,\theta)$ are the acquired k-space data points for projection θ. The PA of each projection in the sinogram can be calculated and used to reconstruct the image via back-projection. Phantom data (3T Philips Intera, SE sequence, TE/TR=10/400ms, FoV=200mm, 5mm slice, 256 projections with 81 frequency encoded points) of two concentric cylinders was used to test the methodology. Images of the brain, heart and calf were also reconstructed (calf shown below: 3T Philips Intera, FSE sequence, TE/TR=60/1000ms, FoV=390mm, 4mm slice, 256 projections, 181 frequency-encoded points) to assess the performance of the method *in vivo*. Fully sampled test data was acquired with either a radial or a Cartesian trajectory so as to have high resolution reference images. When Cartesian data was employed, a radial acquisition was simulated using the Radon transform. In all cases radial data lines were then truncated in k-space to assess the performance of the PA on truncated data sets.

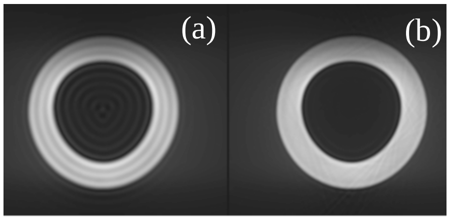

Figure 1: The phantom image reconstructed via radial Padé (b) has improved spatial resolution over the Fourier reconstruction (a) with the same data and the truncation artefact visible in the Fourier reconstruction is removed by the Padé reconstruction.

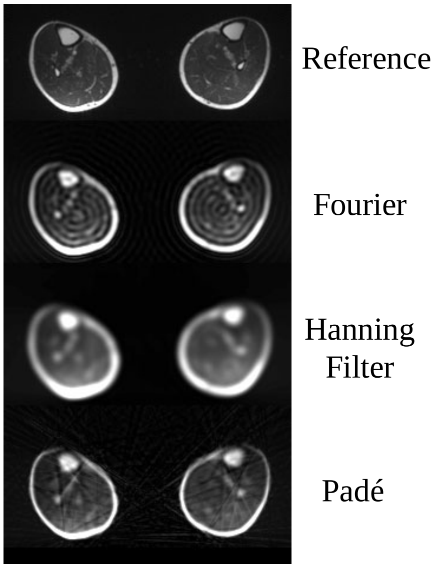

Figure 2: The Padé methodology can be used to give improved reconstructions of *in vivo* images. A hanning filter removes the truncation artefact visible in the Fourier reconstruction but with a severe loss in spatial resolution. The Padé reconstruction both removes the truncation artefact and preserves spatial information.

Results: The extrapolative properties of the FP reconstruction leads to improved spatial definition in comparison to the conventional Fourier reconstruction with the same input data (see figures). Truncation artefact is completely removed from the Padé reconstruction because the PA represents the object by a non-truncated data set. Isolated errors in the approximation of the fully converged signal intensity of the sinogram can occur with highly reduced data sets. These lead to distributed low signal intensity in the reconstructed image along directions corresponding to the projection angle in which the error occurred. The artefact appears quite benign and is counter balanced by the complete removal of truncation artefact from the reconstruction. As more data is included, the level of artefact is reduced because of improved convergence acceleration for the PA.

Discussion: In the Cartesian approach, Padé images reconstructed from reduced data sets suppress texture when insufficient data has been included to capture finer details. The strength of Cartesian Padé reconstruction lies in feature extraction because of the improved structure definition that can be achieved at edge points through convergence acceleration. In the radial case, the trade-offs appear to differ, with the strength of the Padé method lying in improved image reconstruction. This is attributed to the fact that the PA now approximates a projection through the entire object in which the signal intensity tends to vary smoothly rather than approximating individual lines of an image in the phase-encoded direction, in which particular edges may dominate. Another application of this approach is to combine Padé reconstruction with a spiral acquisition trajectory for dynamic studies. In a single shot the centre of k-space can be densely sampled, but only to a limited radial extent. Reconstruction from the methodology described above could result in a rapid single shot acquisition with improved structural resolution.

References:

1. Padé Approximants and their Applications. Ed: Graves-Morris, P.R., Academic Press, 1973.
2. Smith, M.R., *et al.*, IEEE Med Imaging, 1986. MI-5(3): p. 132-39
3. Belkic D. Nucl Instrum Meth A 2001;471(1-2):165-169

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