Estimation of Thermal Noise in the Presence of Artifacts in EPI Images

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Introduction: In the presence of noise, the image intensity in magnitude MRI is governed by a Rician distribution. In a signal-free region (outside an image object), thermal noise is transformed into Rayleigh distribution [1] although the thermal noise has a standard distribution. In practical operation, the estimation of thermal noise in a magnitude image is obtained by multiplying noise intensity with a correctional factor of $\sqrt{2-\pi/2}$ (about 0.655) to get the genuine standard deviation of noise [2]. However, such an operation could overestimate the thermal noise level when there are imaging artifacts. In the present study, we propose a new method to estimate the thermal noise in the presence of image artifacts. By analyzing real and imaginary channels of image data without prior knowledge of artifacts, the noise intensity can be well estimated. We further demonstrate its application to the evaluation of thermal noise in gradient-echo EPI images employed for fMRI studies.

Theory: In signal-free regions, reconstructed imaging datasets of the real and imaginary channels have three variable components: intensity $\boldsymbol{a(t)}$, phase $\boldsymbol{\theta(t)}$, and thermal noise $\boldsymbol{n(t)}$, which can be expressed as follows: $R(t) = a(t)\cos(\theta(t)) + n_1(t)$; and $I(t) = a(t)\sin(\theta(t)) + n_2(t)$, (1)

assuming the artifacts are far smaller as compared to the signal, and their intensities are constant, denoted as level $\boldsymbol{a}$. The phase of these artifacts can be expressed as $\theta(t) = \overline{\theta(t)} + \Delta\theta(t) = \theta + \Delta\theta(t)$; the fluctuation of phase $\Delta\theta(t)$ is relatively small as well. Then Eq. (1) can be modified as:

$$R(t) = a\cos(\theta(t)) + n_1(t) \approx a(\cos\theta - \sin\theta\cdot\Delta\theta(t)) + n_1(t);\quad I(t) = a\sin(\theta(t)) + n_2(t) \approx a(\sin\theta + \cos\theta\cdot\Delta\theta(t)) + n_2(t). \tag{2}$$

The standard deviation of phase fluctuation is denoted as σ_θ, and the noise in the two channels is mutually independent and can be modeled as zero mean with standard deviation σ_0. Thus the thermal noise intensity σ_0 can be solved through the one of following equations:

$$Var(R(t)) = a^2\sin^2\theta\cdot\sigma_\theta{}^2 + \sigma_0{}^2;\quad Var(I(t)) = a^2\cos^2\theta\cdot\sigma_\theta{}^2 + \sigma_0{}^2, \tag{3}$$

where *Var*() denotes the variance. The artifact level $\boldsymbol{a}$ and phase expectation $\boldsymbol{\theta}$ can be estimated from:

$$a^2 = \left(\overline{R(t)}\right)^2 + \left(\overline{I(t)}\right)^2 \quad\text{or}\quad \theta = \arctan\left(\overline{I(t)}/\overline{R(t)}\right). \tag{4}$$

While the Rician model only holds for normal distribution, the proposed method from the solution of Eq. (3) has no restriction on the types of distributions of thermal noise. It is known that the estimation of the phase θ is dependent on the signal-to-noise ratio (SNR), and the solution of Eq. (3) can give a reliable estimation of σ_0, which has been proven (data not shown).

Materials and Methods: Three types of noise estimation methods are employed in the datasets with different SNRs (α/σ_0). One method is to directly use the raw variance of voxel time courses (defined as the ***Raw*** method); another is to correct with a distribution transform factor of 0.655 (called the ***Rayleigh*** method); the third is to get the solution from Eq. (3) (called the Complex Model Solution method, ***CMS***). A single-shot gradient-echo EPI sequence was employed with the following imaging parameters: TR of 2 s, TE of 40 ms, FOV of 24 cm, matrix of 64×64, slice thickness of 6 mm. Imaging slices were acquired and repeated 100 times to obtain the voxel time courses. Informed consents were obtained from all subjects. All subjects were in resting status during the scanning. A signal-free region at the upper-left corner of the images is selected to estimate the noise with a lower artifact level than that from global voxels. Those voxels that have estimated phase fluctuations under 0.5, and artifact levels below 1/20 of the signal intensity (mean values of phantom or brain region) are selected to calculate global noise intensity.

Results and Discussion: The simulation results are shown in Fig. 1. As expected, when the SNR is very low (corresponding to the signal-free region), the ***Raw*** method underestimated the noise intensity, and both the ***Rayleigh*** and the ***CMS*** methods made a correct estimation. However, the ***Rayleigh*** method overestimated the noise level when the SNR is larger than 1. The simulated ***CMS*** method always gives a correct estimation no matter how much the SNR is as shown in Fig. 1 (left panel). When using experimental data to calculate the noise levels using the three methods, the estimations of noise level are SNR dependent. As predicted from the simulated curve, at a low SNR (phantom) both the ***Rayleigh*** and ***CMS*** methods provided a correct value and the ***Raw*** underestimated. At SNR close to 1 (in the case of the upper-left corner), the ***Rayleigh*** overestimated, the ***Raw*** underestimated, and the ***CMS*** correctly estimated the noise level. At the higher SNR of 3 (global voxels), the ***Rayleigh*** method continuously overestimated the noise level whereas the ***Raw*** and ***CMS*** methods provided correct estimations. These results demonstrated that when images are contaminated by artifacts, the ***CMS*** method provides a reliable estimation of the noise level with robustness at different SNRs. In conclusion, the proposed method can adapt to more general situations (with or without artifacts) for noise intensity evaluation.

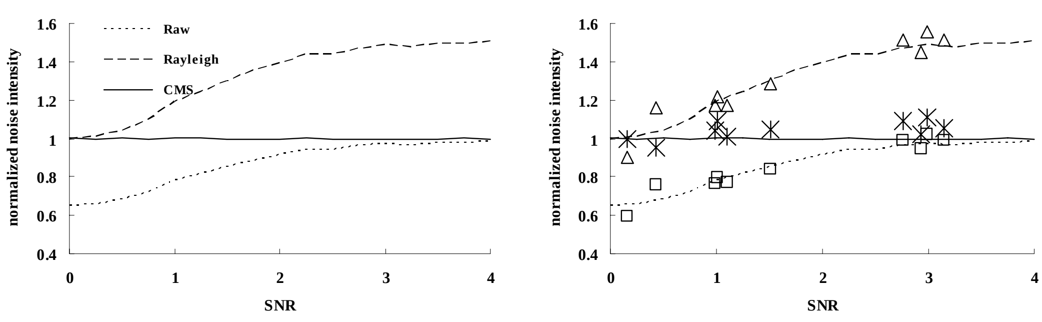

Fig.1. (left) The dependence of noise estimation on SNR by using three different methods with the simulated data. (right) The noise estimation with experimental data at different noise levels with ***Raw*** (square), ***Rayleigh*** (triangle), and ***CMS*** (star) methods. Both panels have the same x-axis for SNR and y- axis for estimated noise intensity.

References: 1. Hákon Gudbjartsson et al., Magn. Reson. Med., 34:910-914, 1995. 2. Mark Haacke et al., *Magnetic Resonance Imaging*, 1999.

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